What can geotechnical engineers learn from granular mechanics?

Malcolm Bolton

Justifying my agenda

- “Granular mechanics” is probably why you are here.
  - It is satisfying to explain complex material behaviour
  - There is real scope for making breakthroughs
  - There is a growing international community
  - So there are good journals, nice conferences…
- But geotechnical engineering?
  - It is typical, and frustrating, to find minimal soil test data
  - Professional conservatism can strangle new ideas
  - Codes, unlike science, seem shackled to national borders

Geotechnical engineering

- What is the problem?
  - Soil is fundamentally non-linear and changeable, but…
  - …soil mechanics textbooks deal in constant parameters…
  - …which leads to uncertainty in fixing their values…
  - …and the tendency to ignore test data…
  - …in favour of traditional, conservative values…
  - …protected by traditional, conservative safety factors…
  - …which divorce designers from reality…
  - …and make predictions of performance very uncertain…
  - …leading to waste, costs and delays in construction projects.
- So what is the answer?

Dealing with non-linearity

- Granular mechanisms are needed to validate new parameters that capture the inherent variability of "constants" such as c', ϕ', E, ν, m_v, k, C_v and create indelible impressions of the causes of non-linearity in the minds of future geotechnical engineers.
- But we must be realistic about what we can teach students, and what test data we demand from project engineers – which will not be of the quality and quantity we use ourselves to construct theories.
- I will propose the efficient use of databases, dealing first with strength, and then with stiffness.

Strength: the background

- Most geotechnical engineers are taught to assume a linear Mohr-Coulomb envelope of peak strength, with parameters c', ϕ': $\tau_{max} = c' + \sigma' tan\phi'$
- But the strength envelope is non-linear, and is also a function of density which varies from point to point.
- It is bad practice in risk analysis to use two parameters, each varying with stress and density, when a single variable (secant ϕ) will do the job.
- Can DEM simulations in support of Cam Clay, Stress-Dilatancy, soil databases, and best-practice QRA finally persuade engineers to ditch $c'\phi'$?

"Empirical" study of strength and dilatancy

- Bolton (1986) used the data of triaxial and plane strain tests on a total of 17 sands to correlate the peak component of the secant angle of internal friction $\Delta\phi_{dil} = (\phi_{max} - \phi_{crit})$ with the angle of dilatancy ψ_{max}, and with a new Relative Dilatancy Index I_R.
- This emphasises the importance of two fundamental soil parameters, ϕ_{crit} which is the angle of friction at constant volume, and a new parameter σ_c which was used to normalise the confining stress σ' at failure with regard to its effect of suppressing dilation.

Relative dilatancy index I_R

- Relative dilatancy index $I_R = I_D I_C - 1$
- Relative density $I_D = (e - e_{min}) / (e_{\max} - e_{min})$
- Relative crushability $I_C = \ln(\sigma_c / \sigma')$
  - σ_c is the aggregate crushing stress, at which the 1D compression curve for dense soil joins the ncl.
  - Typical values: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.
  - σ' is the effective stress normal to a shear plane (e.g. in the SSA).
  - In triaxial tests, σ_c can be replaced by an identical value for p_c, and σ' can be replaced by p'.

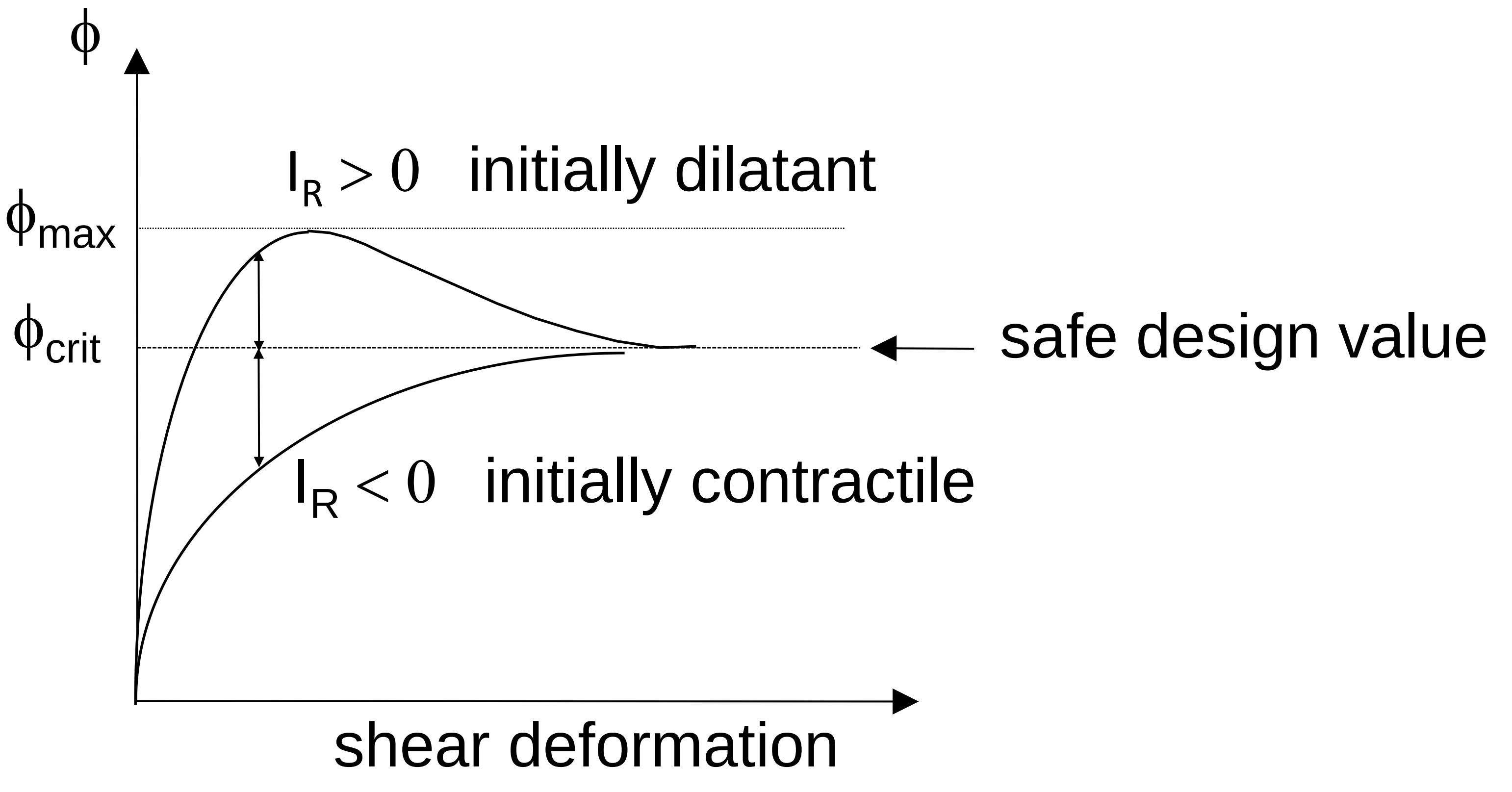
Strength and dilatancy correlations with I_R

- The following statistical correlations are available:
  - plane strain (e.g. SSA) conditions

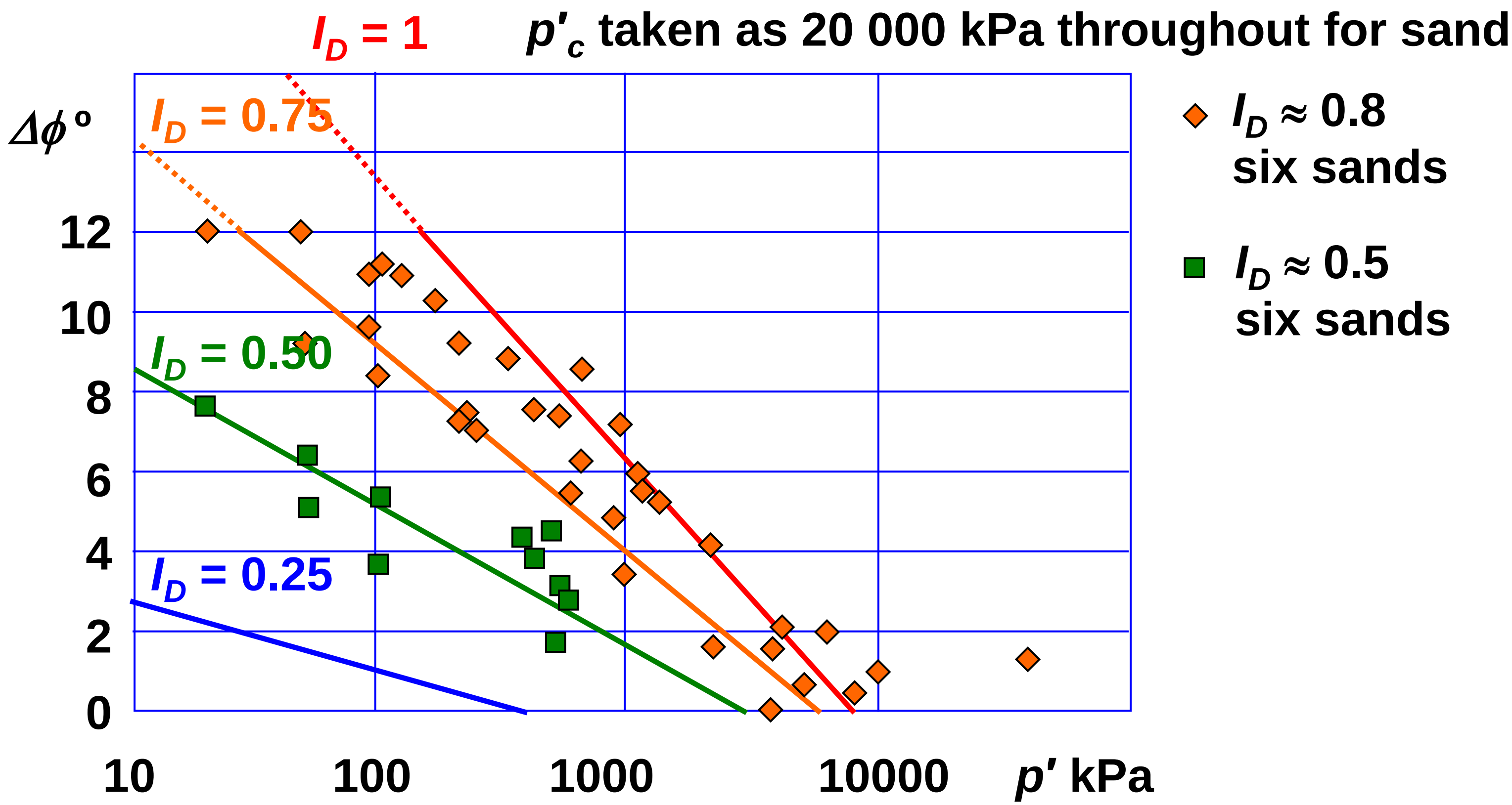
    $(\phi_{max} - \phi_{crit}) = 0.8\ \psi_{max} = 5\ I_R$ degrees
  - triaxial strain conditions

    $(\phi_{max} - \phi_{crit}) = 3\ I_R$ degrees; $(-\delta\varepsilon_v / \delta\varepsilon_1)_{\mathrm{max}} = 0.3\ I_R$
- Qualitatively similar to Andrew Schofield's Cam Clay work equation, and to Peter Rowe's Stress-Dilatancy Theory, but now calibrated for magnitudes.
- Use with caution when $I_R > 4$, due to sparse data.
- Influence of bedding on anisotropy: DSS vs. Triaxial.

Dilatancy contribution to internal friction

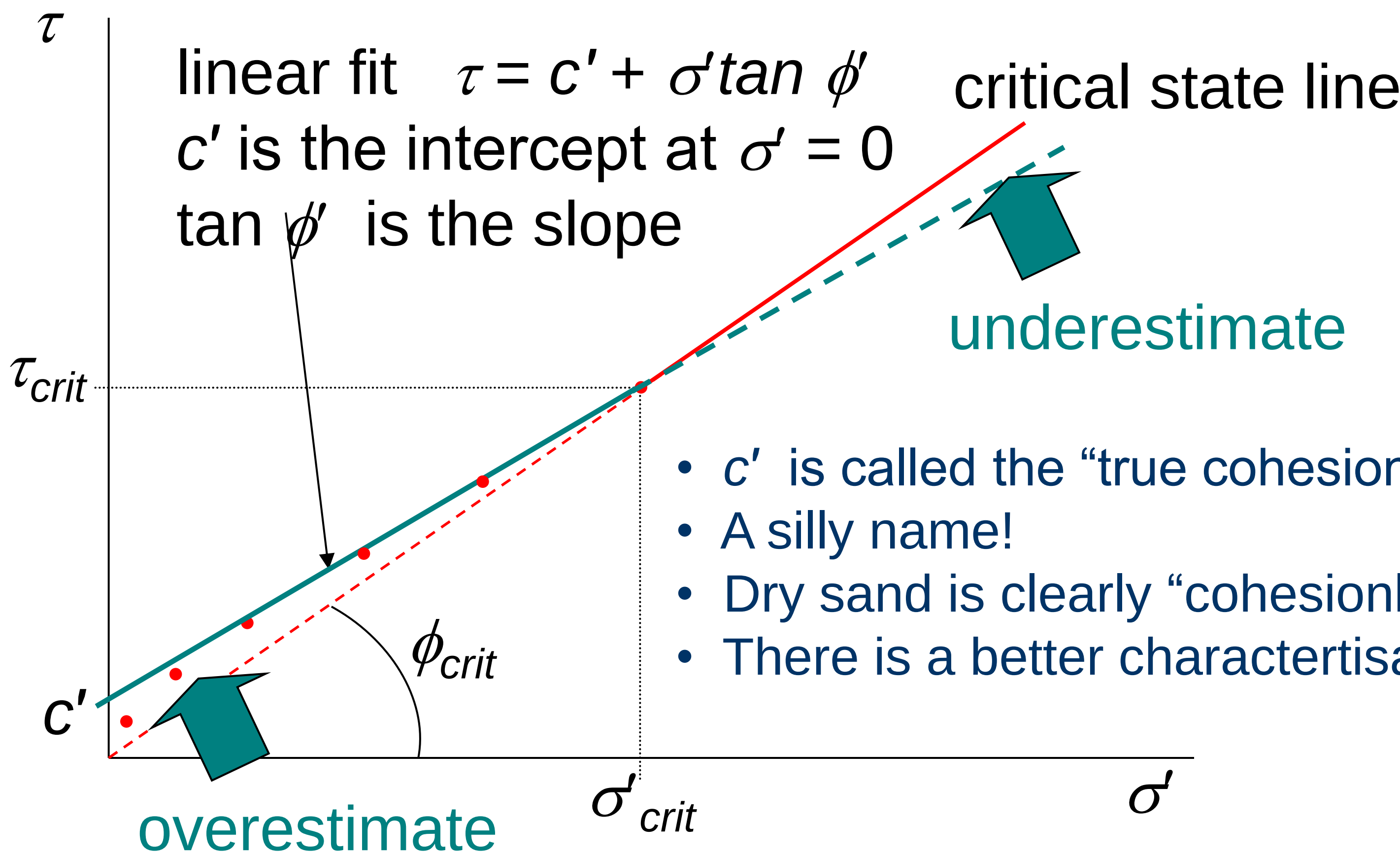


Success of correlations: triaxial $\Delta\phi = 3\ I_R$ for 6 sands



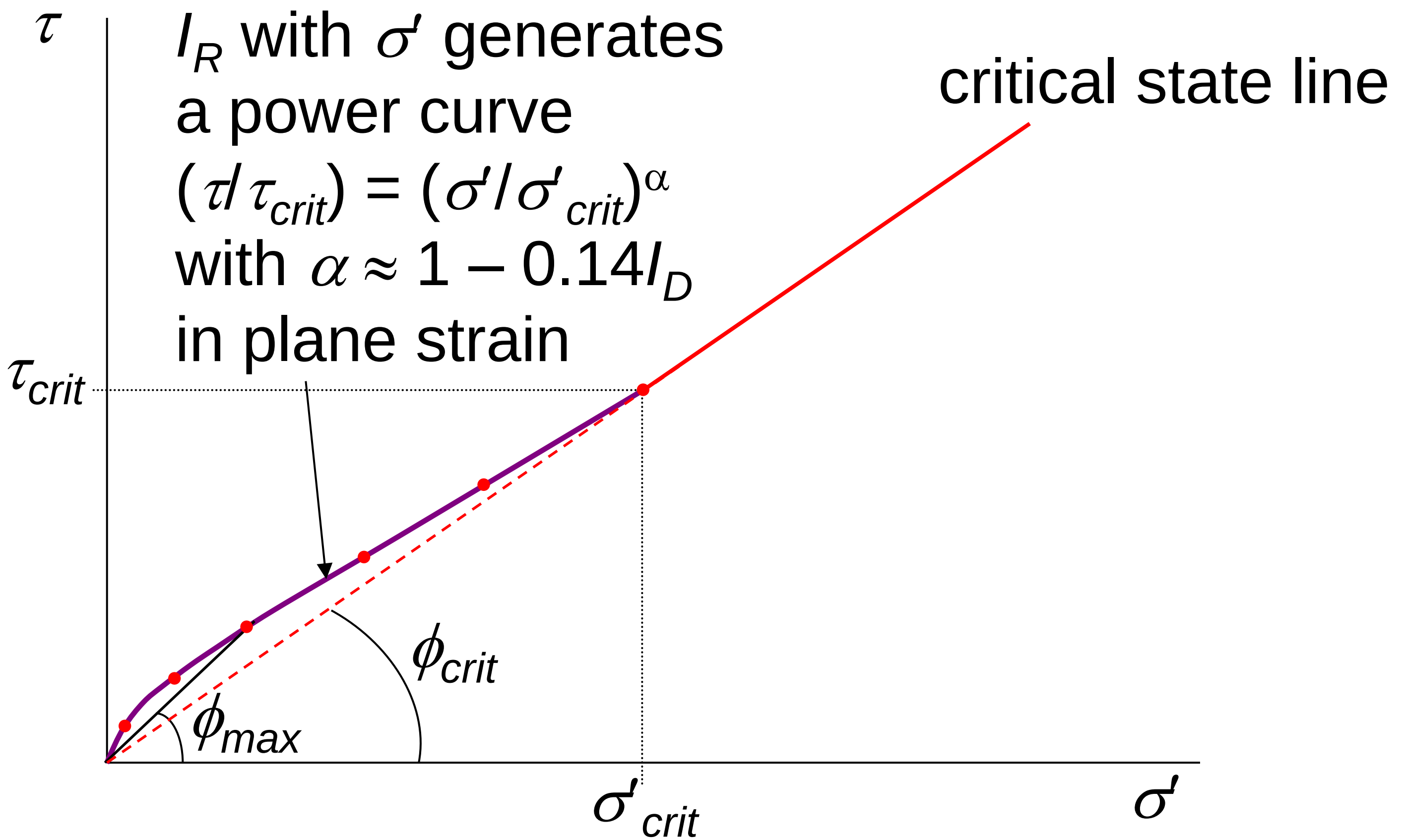
Bolton (1986) Geotechnique

Linear M-C strength envelope for a given density

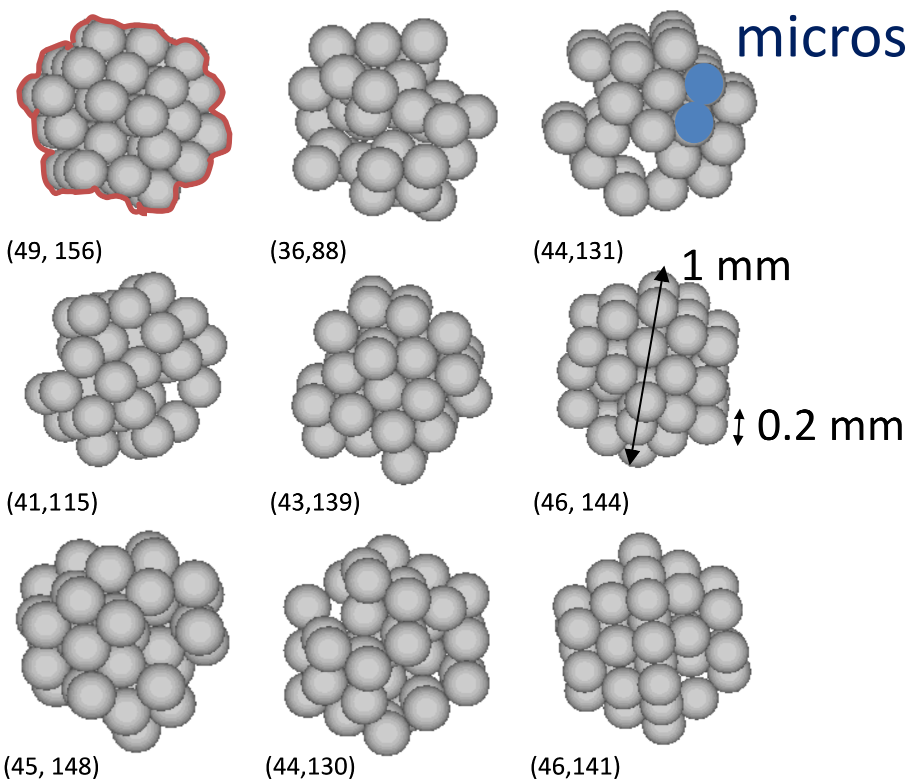


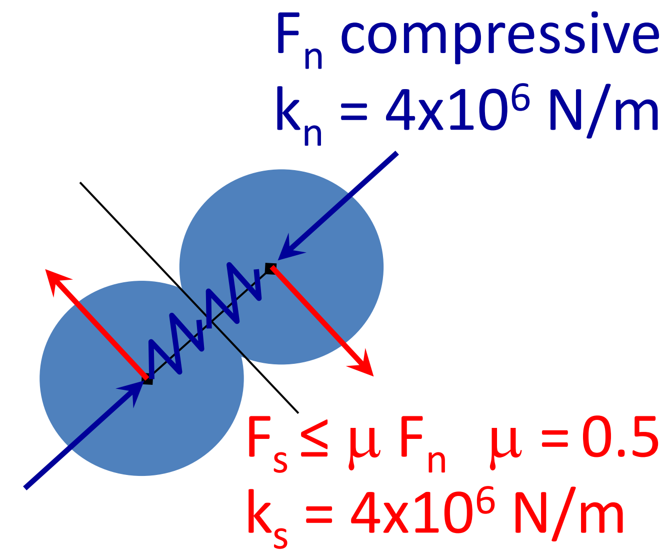
- *c′* is called the "true cohesion" intercept
- A silly name!
- Dry sand is clearly "cohesionless"
- There is a better charactertisation…

Curved M-C strength envelope at a given density



Agglomerates: discrete models of sand grains



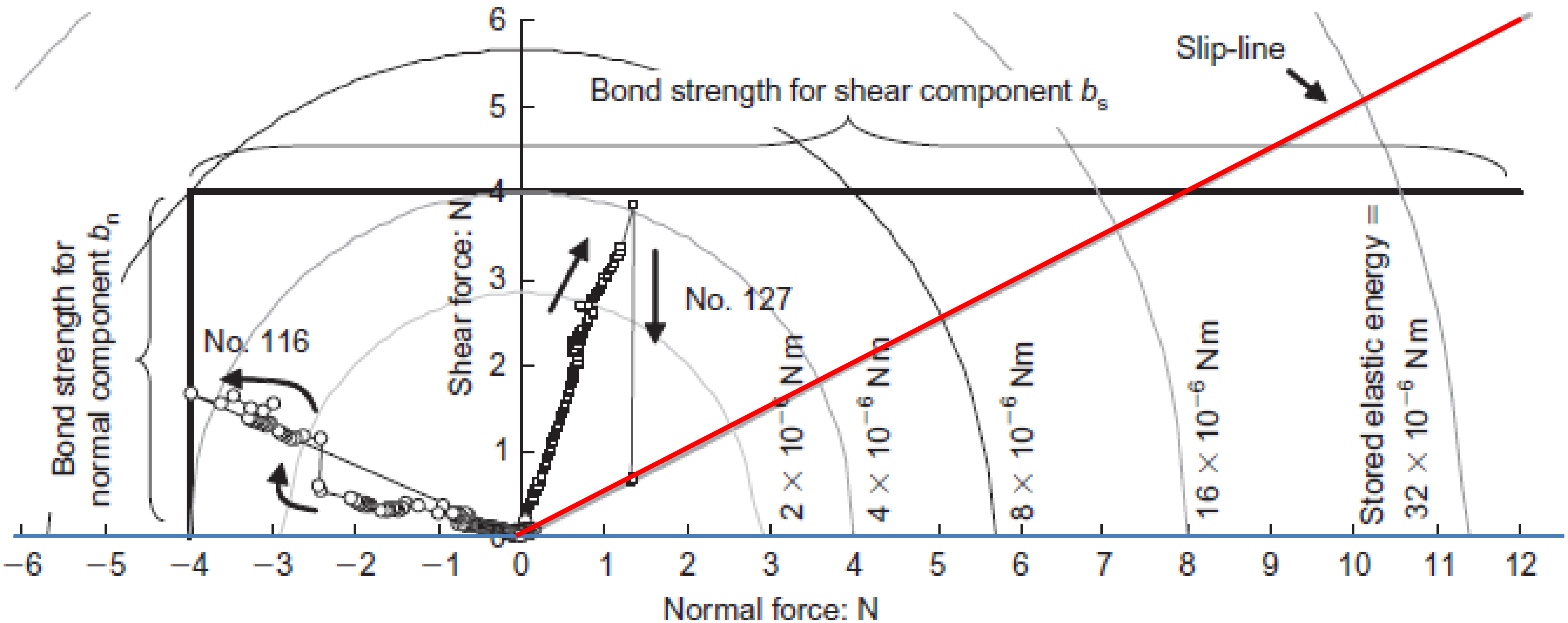


contact bond $F_b = T_b = 4$N

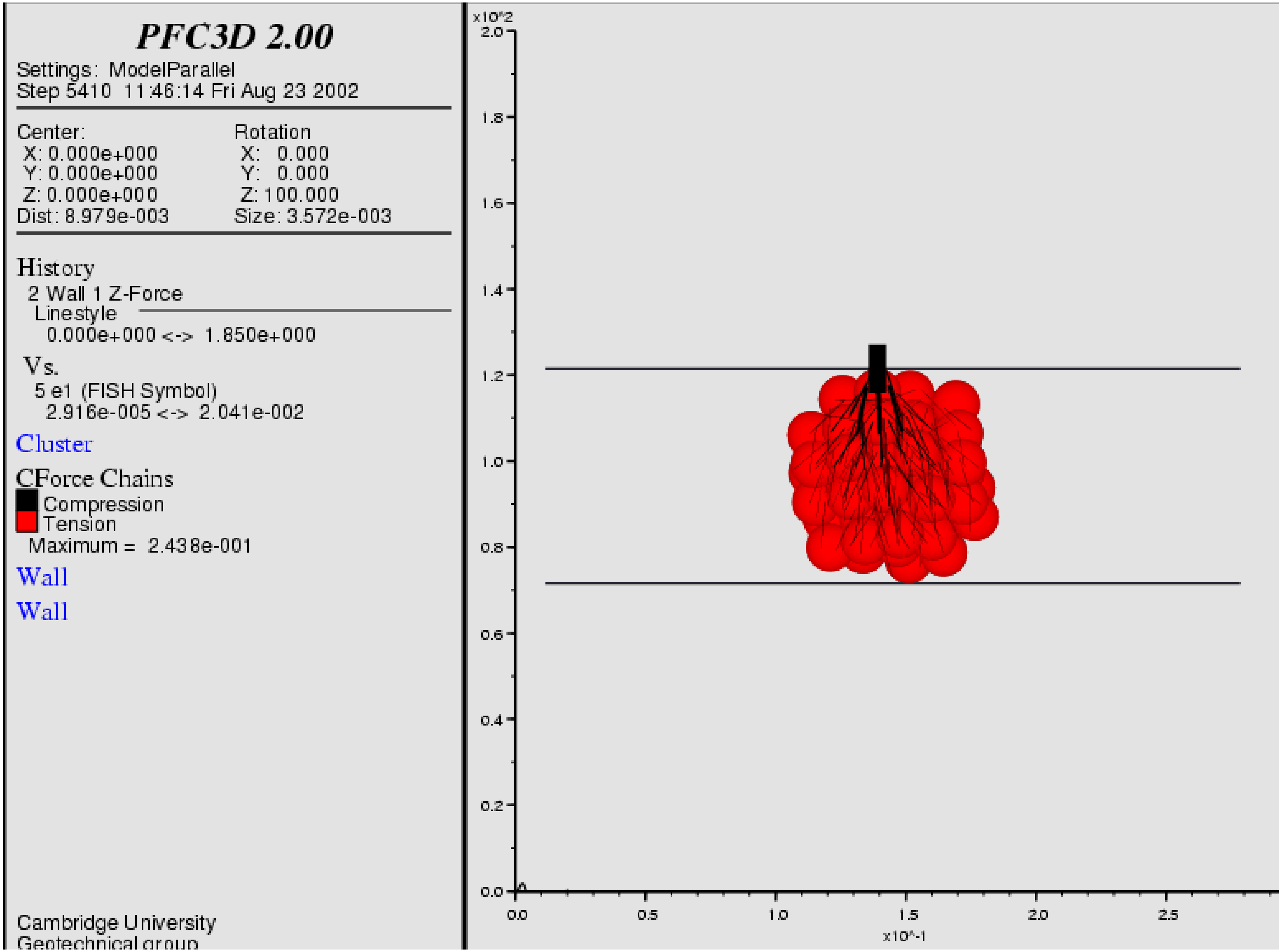
Perfect “crystal” agglomerate has 57 micro-spheres 0.2 mm diameter held together by 228 bonds as strong as rock. Then random flaws are introduced.

Robertson (2000) PhD; Cheng, Nakata & Bolton (2003) Geotechnique

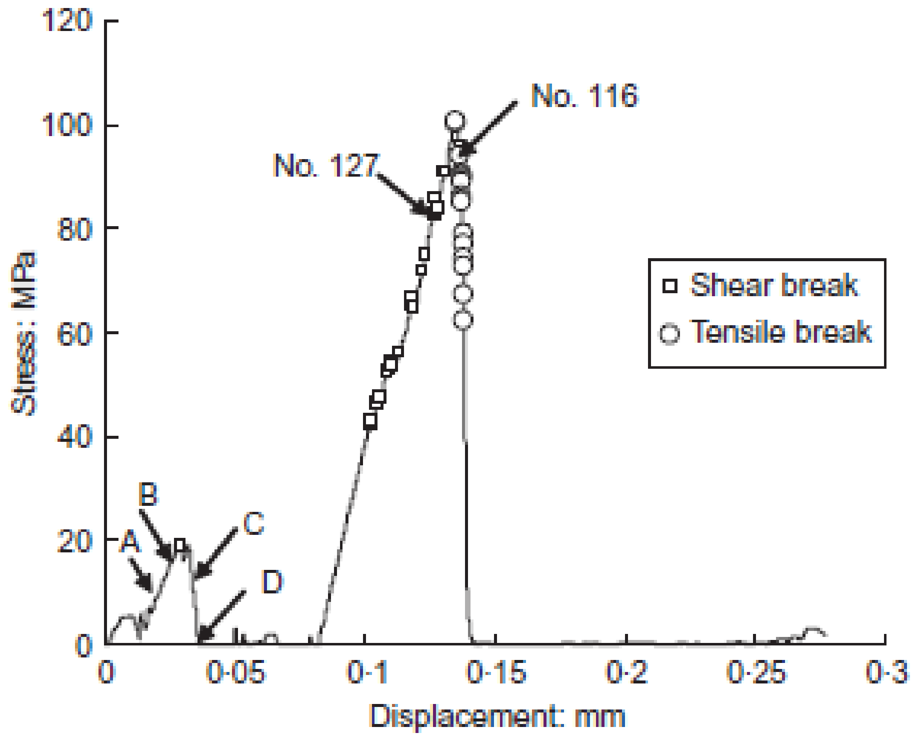
Modelling a bonded contact in DEM

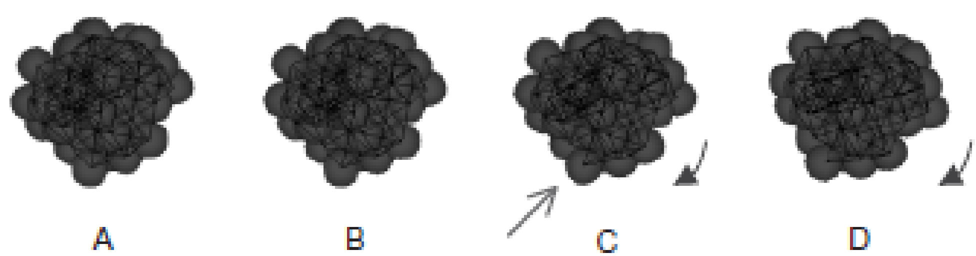


Bolton, Nakata & Cheng (2008) Geotechnique

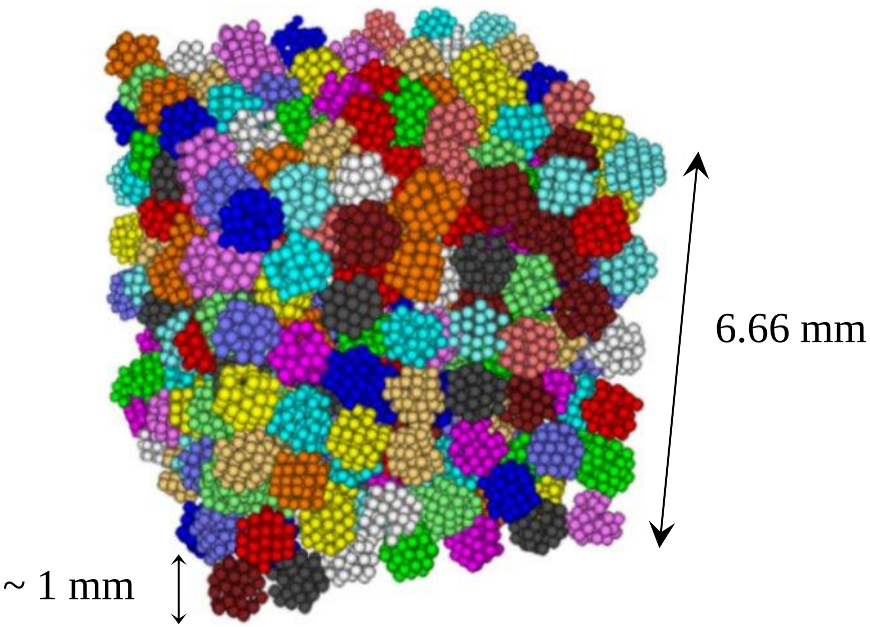

PFC3D 2.00
Settings: ModelParallel
Step 5410 11:46:14 Fri Aug 23 2002
Center:
X: 0.000e+000
Y: 0.000e+000
Z: 0.000e+000
Dist: 8.979e-003
Rotation
X: 0.000
Y: 0.000
Z: 100.000
Size: 3.572e-003
History
2 Wall 1 Z-Force
Linestyle
0.000e+000 <-> 1.850e+000
Vs.
5 e1 (FISH Symbol)
2.916e-005 <-> 2.041e-002
Cluster
CForce Chains
Compression
Tension
Maximum = 2.438e-001
Wall
Wall
Cambridge University
Geotechnical group
x10^2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
x10^-1

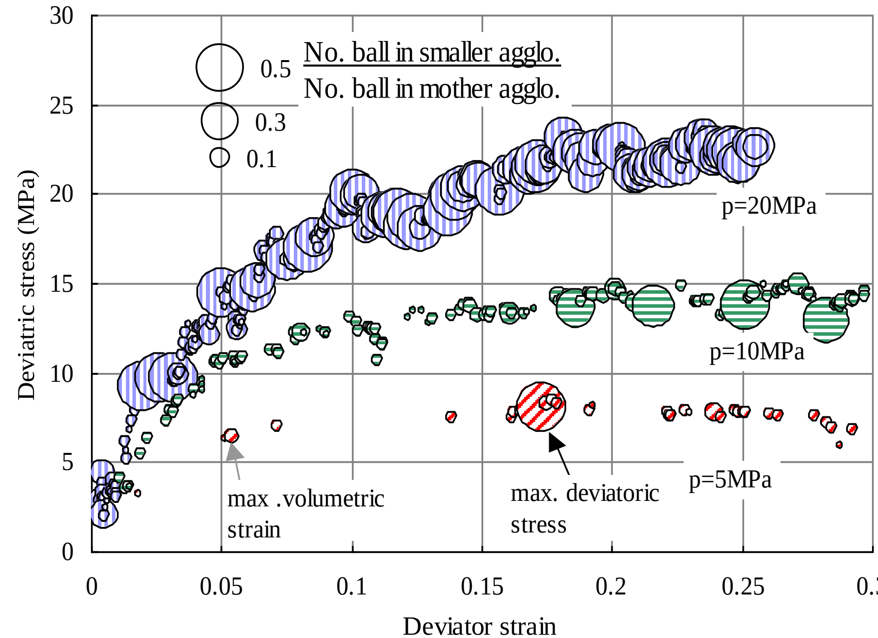
Platen test on an agglomerate

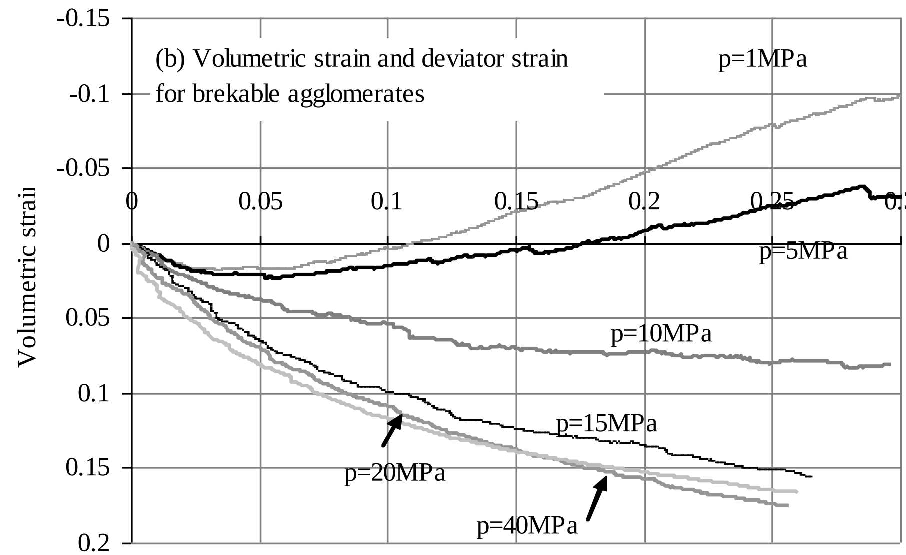




Fragmentation in constant p′ triaxial compression

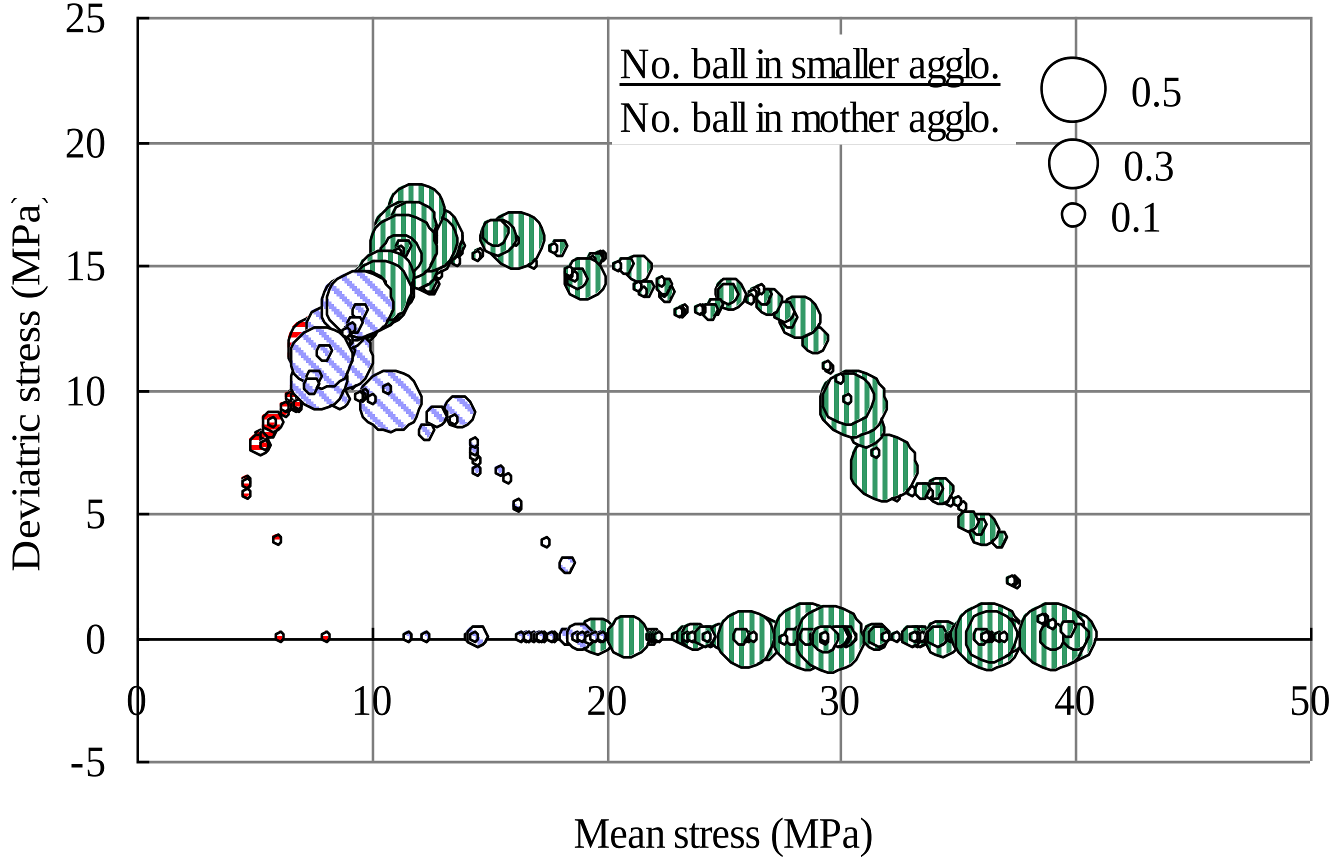




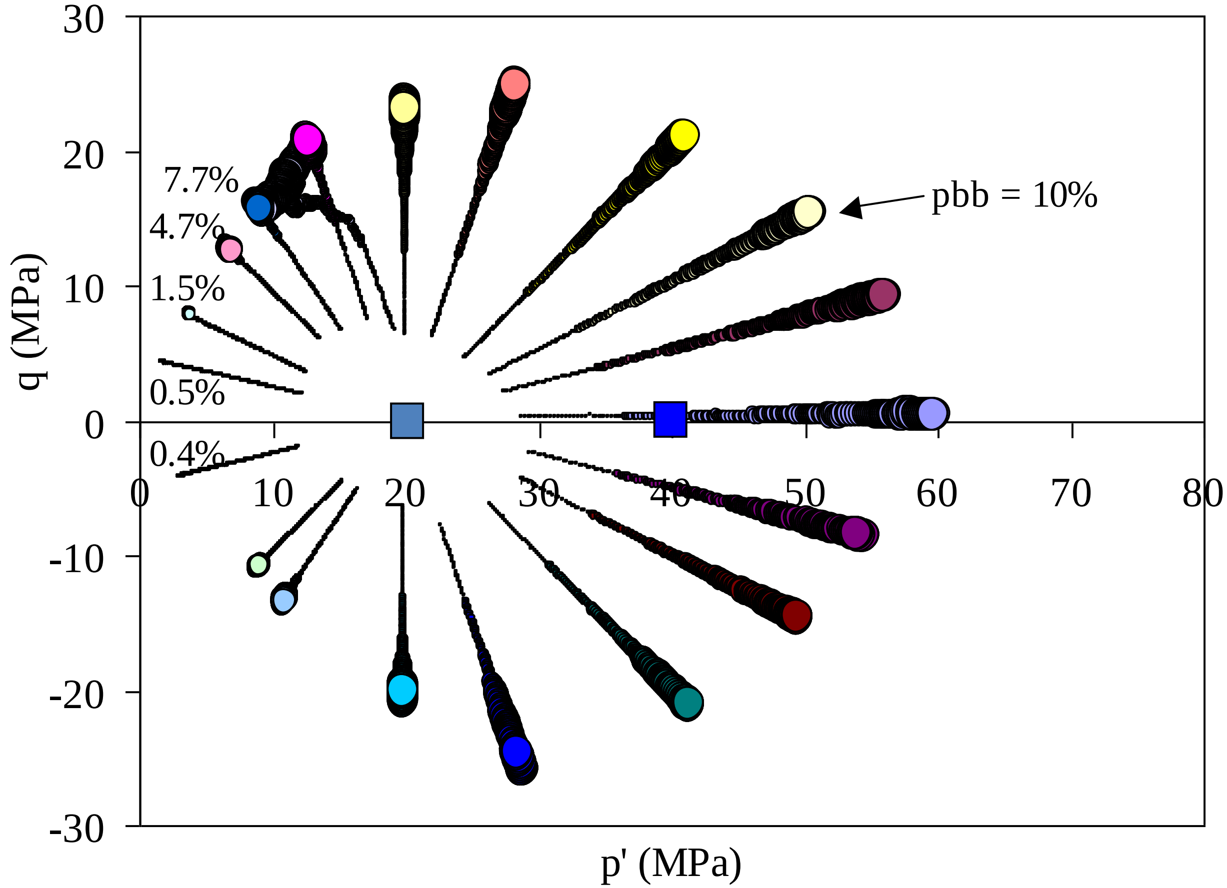


Cheng (2004) PhD

Fragmentation in constant volume triaxial compression



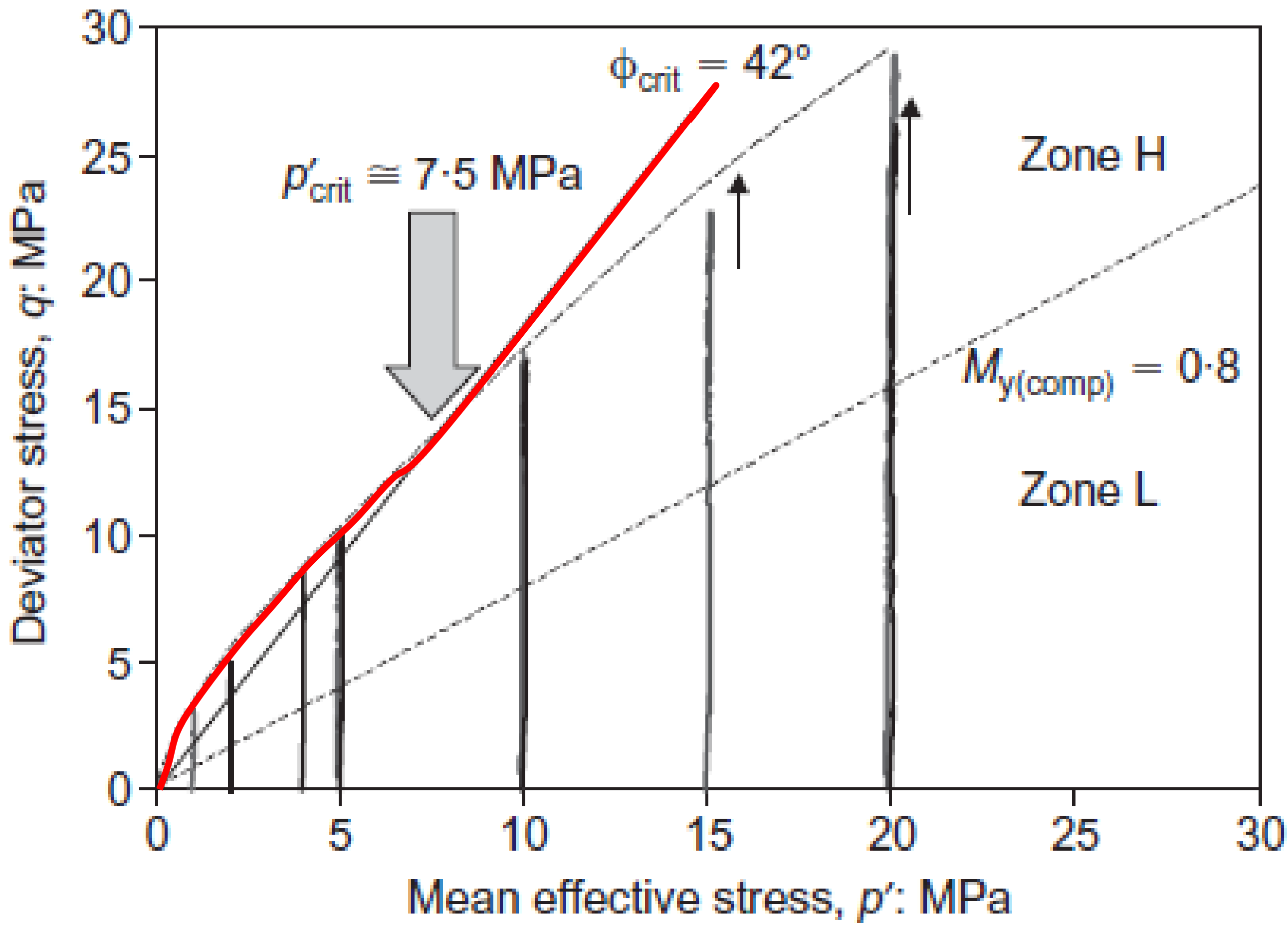
Stress-path tests from OCR = 2



pbb = % of bonds broken from the start of a shearing test

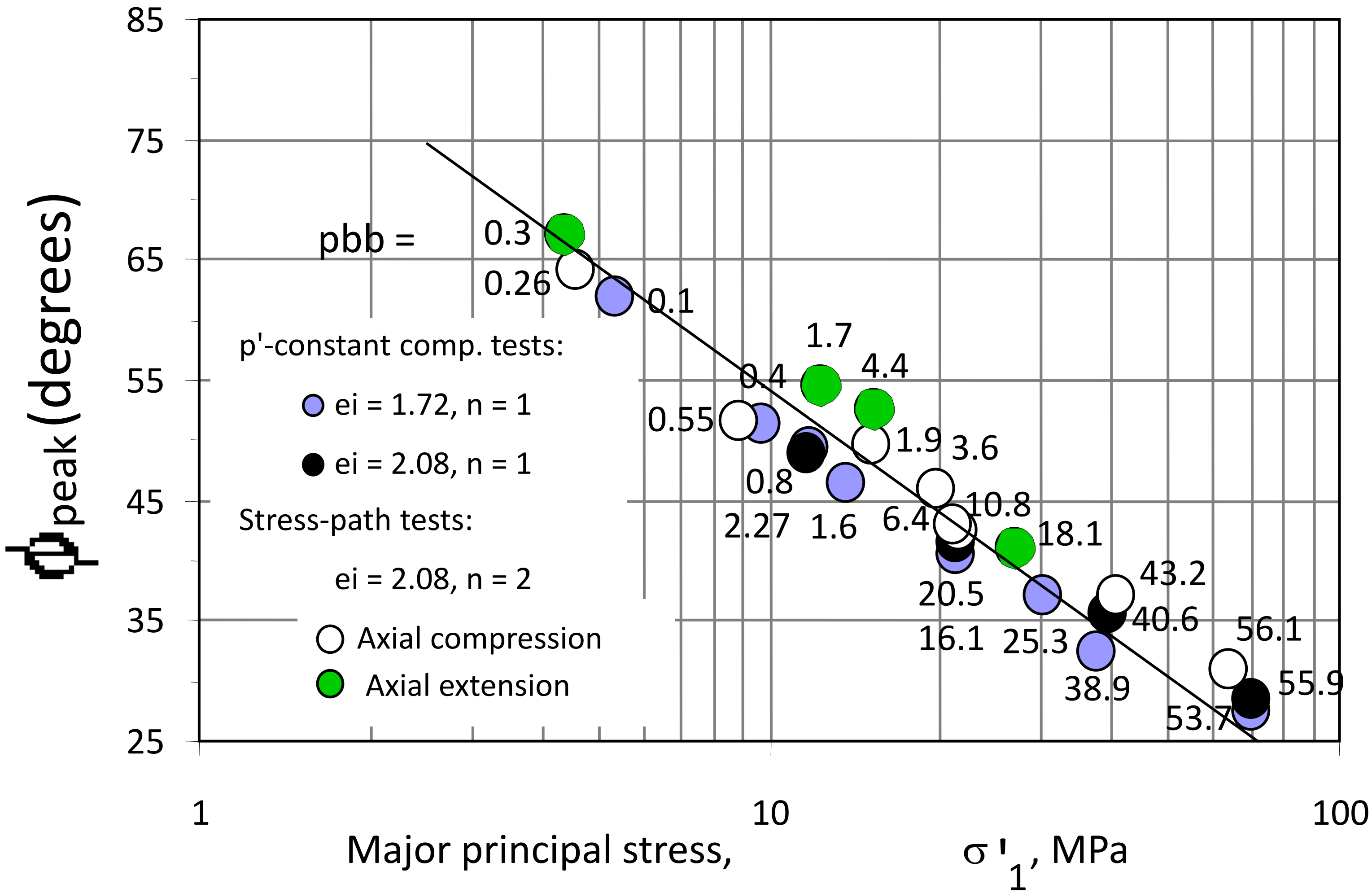
Cheng, Bolton & Nakata (2004) Geotechnique

DEM strength envelope



Cheng, Bolton & Nakata (2004) Geotechnique

Secant ϕ_{max} reducing as a function of $\log\sigma'_1$



Cheng, Bolton & Nakata (2004) Geotechnique

Overview: non-linear strength envelope

- Strength at large strains: use ϕ_{crit}.
- Strength of soils that dilate: use $\phi_{max} = \phi_{crit} + \Delta\phi_{dil}$ where $\Delta\phi_{dil} \approx 0.8\,\psi_{max}$, or Stress-Dilatancy if preferred.
- For clean quartz sands estimate $\Delta\phi_{dil}$ from I_R by using σ_c = 20 000 kPa. Can check σ_c using site sample data.
- Or estimate σ'_{crit} using I_R = 0 to get $ln\left(\frac{\sigma_c}{\sigma'_{crit}}\right) = \frac{1}{I_D}$ so in a DSS test $(\tau/\tau_{crit}) = (\sigma'/\sigma'_{crit})^{\alpha}$ with $\alpha \approx 1 - 0.14 I_D$
- Engineers with enhanced soil classification data can construct their own non-linear strength envelope.
- Ideas and mechanisms broadly validated by DEM.

Empirical?

Stiffness: the background

- Most geotechnical engineers are taught to use elasticity to calculate the settlement of foundations.
- e.g. for a rigid circular base on a uniform, deep bed:

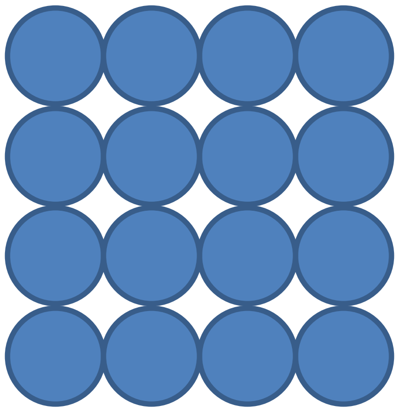
$$\frac{\delta}{D} = \frac{\pi}{4}\left(1 - \nu^2\right)\frac{\sigma}{E}$$

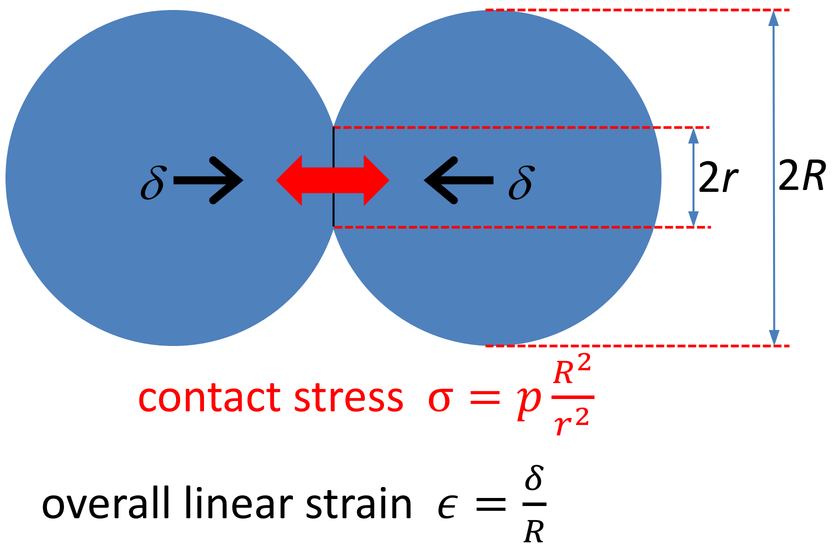
- But soil gets stiffer with mean effective stress (and therefore depth) and less stiff with shear strain.
- Only the best courses show how to pick values of E and ν to suit SI data, and allow for soil non-linearity.
- Most engineers know the elastic formulae but not how to populate them or use them reliably!

Stiffness: contact-hardening #1

Granular contact mechanics – much simplified

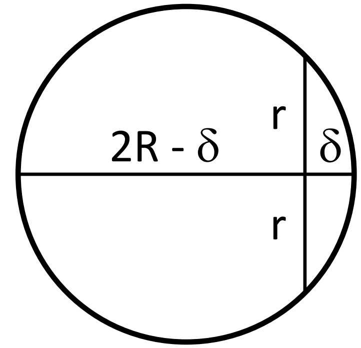
Take a cubical arrangement of spheres under mean effective stress p. Consider the "flat" contact of radius *r* between a pair of spheres of radius *R*, such that *r* << *R*.





After Hertz (1882) etc etc

Stiffness: contact-hardening #2



Chord $2r$ cutting diameter $2R$ into lengths δ and $(2R - \delta) \approx 2R$ gives $r^2 \approx 2R\delta$ by Pythagoras.

Invoking St Venant, a punch of radius r << R on an extensive elastic bed with properties E_g, ν_g for the grain, will indent by

$$\frac{\delta}{2r} \approx \frac{\pi}{4}\left(1 - \nu_g{}^2\right)\frac{\sigma}{E_g}$$

Substituting $r = \sqrt{2R\delta}$ and putting $\sigma = p\frac{R^2}{r^2}$ and $\varepsilon_{vol} = 3\frac{\delta}{R}$ we can derive the volumetric stress-strain relation for the aggregate

$$\varepsilon_{vol}{}^{1.5} \approx 1.5^{1.5}\pi\left(1 - \nu_g{}^2\right)\frac{p}{E_g}$$

Stiffness: contact-hardening #3

Differentiating, we deduce the non-linear bulk modulus

$$K = \frac{dp}{d\varepsilon_{vol}} = \left[\frac{E_g}{\pi\left(1 - {\nu_g}^2\right)}\right]^{\frac{2}{3}} p^{\frac{1}{3}} = C\, p^{\frac{1}{3}}$$

which gives stress-hardening, though not as aggressively as Cam Clay's κ-lines which give $K \propto p$.

And since there is no cross-axis effect in a cubical array, $\nu = 0$.

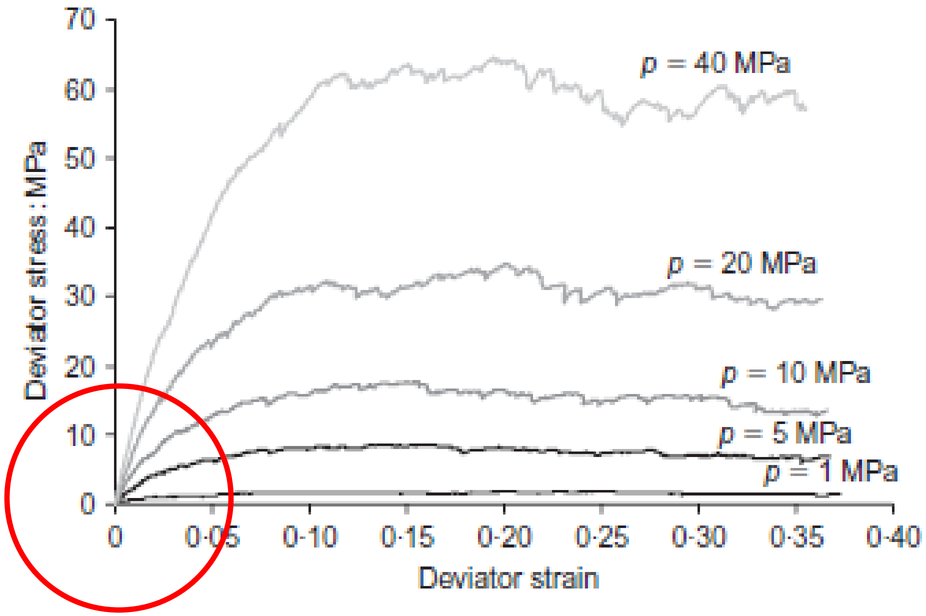
So for a cubical aggregate of linear-elastic spheres, the shear modulus at zero shear strain

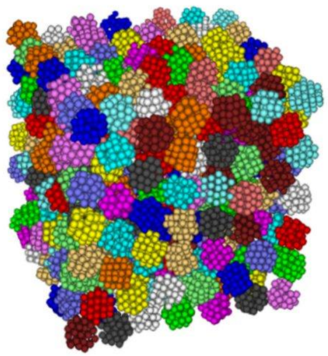
$$G_0 = 1.5K \frac{(1 - 2\nu)}{(1 + \nu)} \approx 1.5 \left[\frac{E_g}{\pi\left(1 - {\nu_g}^2\right)}\right]^{\frac{2}{3}} p^{\frac{1}{3}} = 1.5\, C\, p^{\frac{1}{3}}$$

Stiffness: influence of p'

- Simplified granular mechanics suggests a power law effect of p' on G_0 due to contact hardening.
- But real grains are irregular in shape and packing. And an increase in p' will create more contacts and enhance stiffness independent of contact hardening.
- Denser packings also create more contacts, a smaller average contact stress, and therefore a larger stiffness. So there must also be a density function.
- These propositions can be demonstrated using DEM.

Stiffness: lessons from DEM

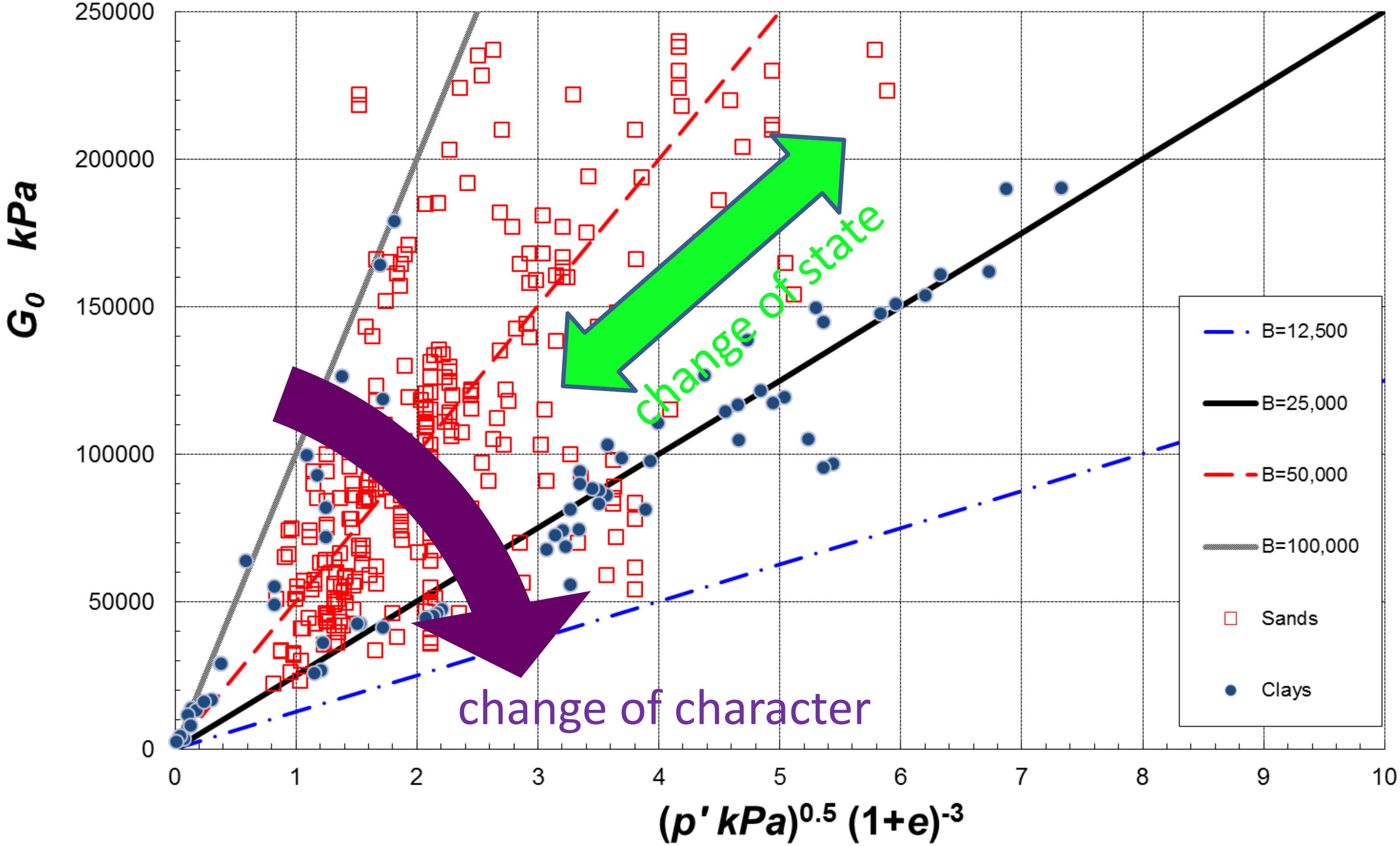




Bolton, Nakata & Cheng (2008) Geotechnique

- Constant-p' triaxial test DEM simulations on an assembly of irregular agglomerates of bonded microspheres with linear springs at their contacts.
- Note that G_0 increases with p' due to grain deformations creating more contacts.
- Real soils should stiffen with p′ via both contacts and fabric.
- We clearly need a database of typical soils.

Database of G_0 : 25 sands/gravels;15 clays/silts



Vardanega & Bolton (2014) JGGE; Oztoprak & Bolton (2013) Geotechnique

Elastic shear stiffness function

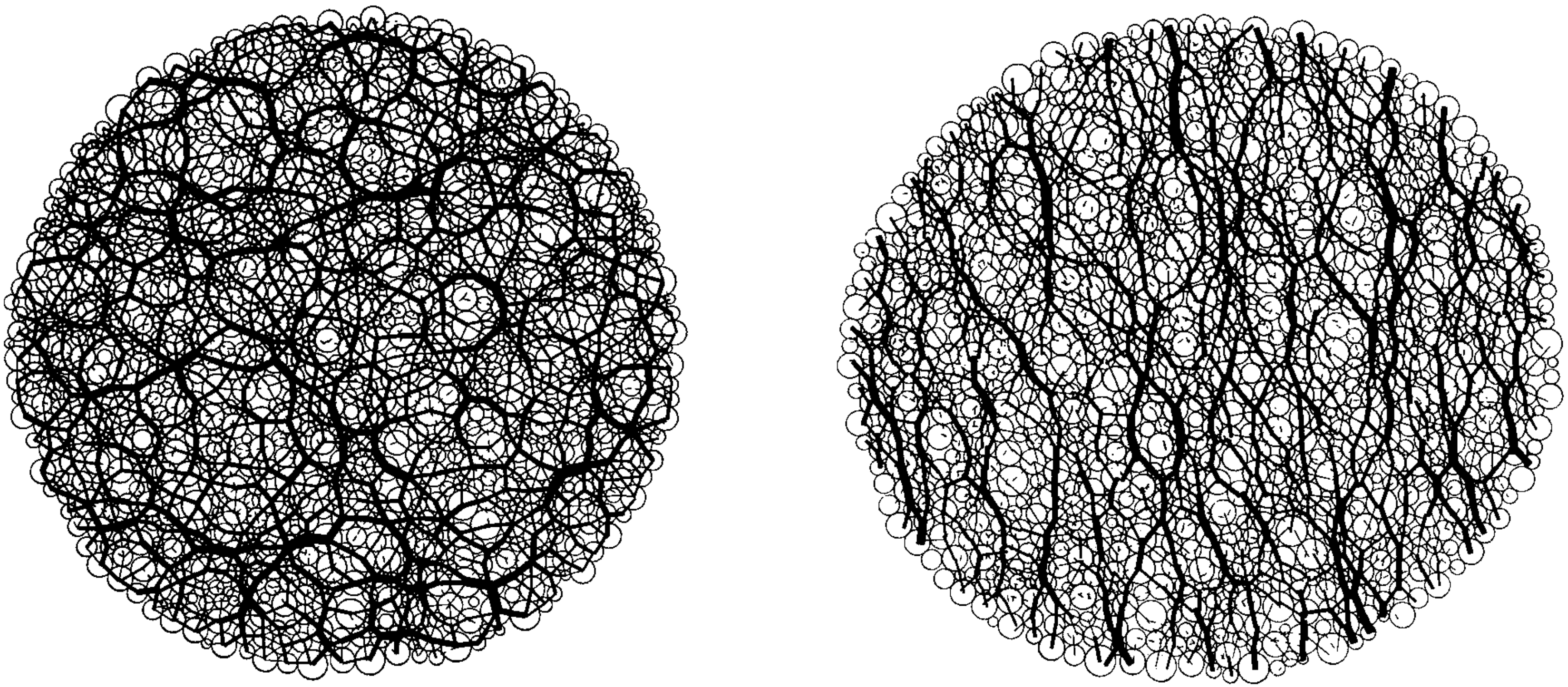
- $G_0 \approx A \frac{(G_g p')^{0.5}}{(1+e)^3} = B \frac{(p')^{0.5}}{(1+e)^3} = C(p')^{0.5}(\rho_d)^3$

  where *A* is a number, but *B* and *C* have dimensions.
- G_0 must be measured at a known *p′* and voids ratio *e* (or dry density ρ_d), and in an appropriate plane of polarization, providing a calibration of *B* (or *C*) for the granulometry and fabric of the soil actually on site.
- Then the engineer can modify G_0 for changes of *p′* that will later occur under a foundation, or an excavation.
- Via granular mechanics, a database, and engineering logic we see that *B* is a material constant, not G_0!

Stiffness: strain-softening #1

- All soils show a reduction of stiffness from their initial G_0 as shear strains γ increase beyond 10^{-5}.
- For monotonic tests, the τ/γ curve is quasi-hyperbolic at least up to $\gamma \approx 10^{-2}$.
- There are relatively small differences between sands (linked to uniformity), and clays (linked to plasticity).
- Both numerical (DEM) and physical (photoelastic) tests link stiffness-reduction with contact sliding and the formation of a "strong contact-force network".

DEM: the evolving contact network

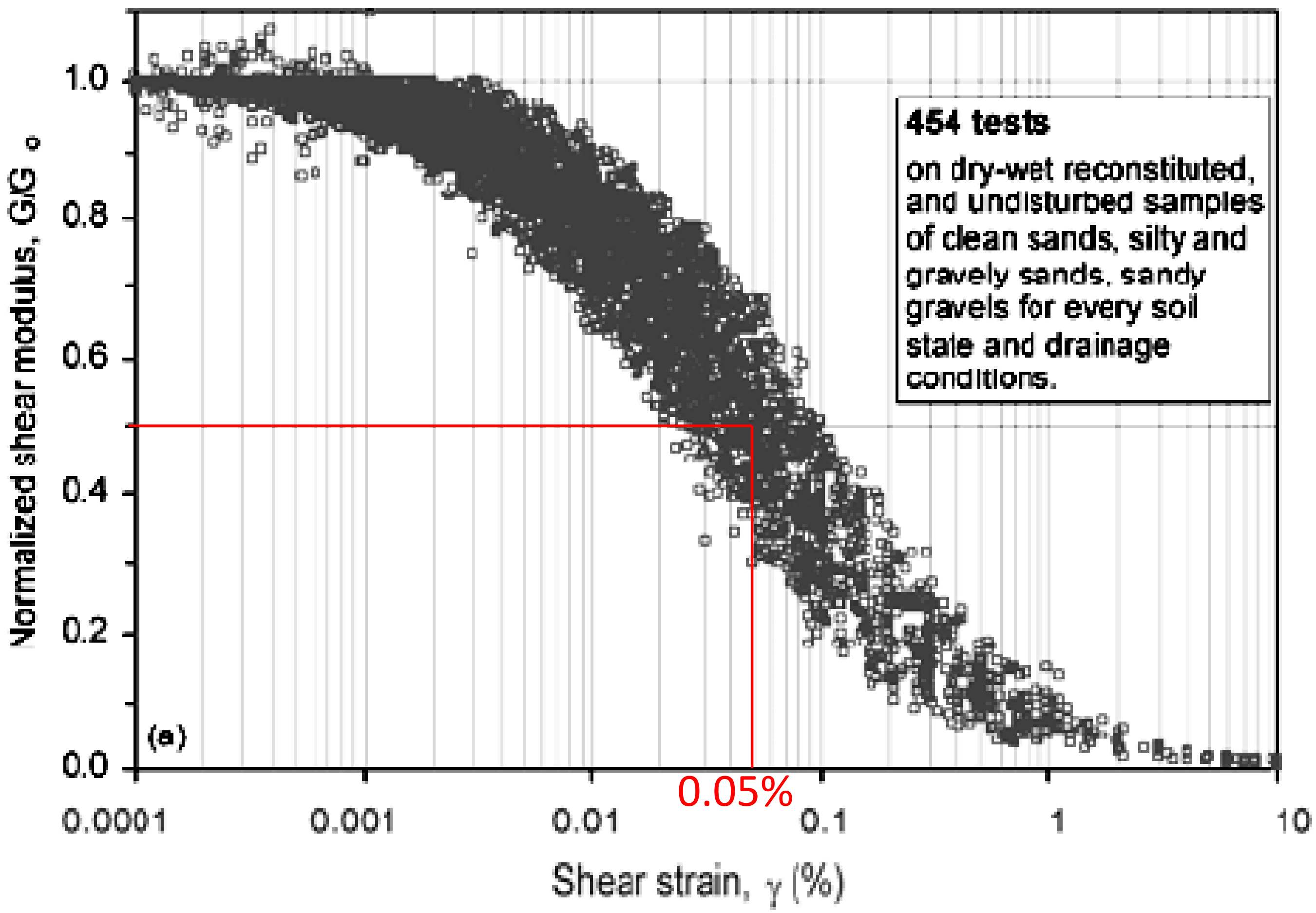


Numerical simulation of contact force distributions : thicker lines, larger force.
(a) under isotropic stress *(b) under vertical compression*
(after Thornton and Barnes, 1986)

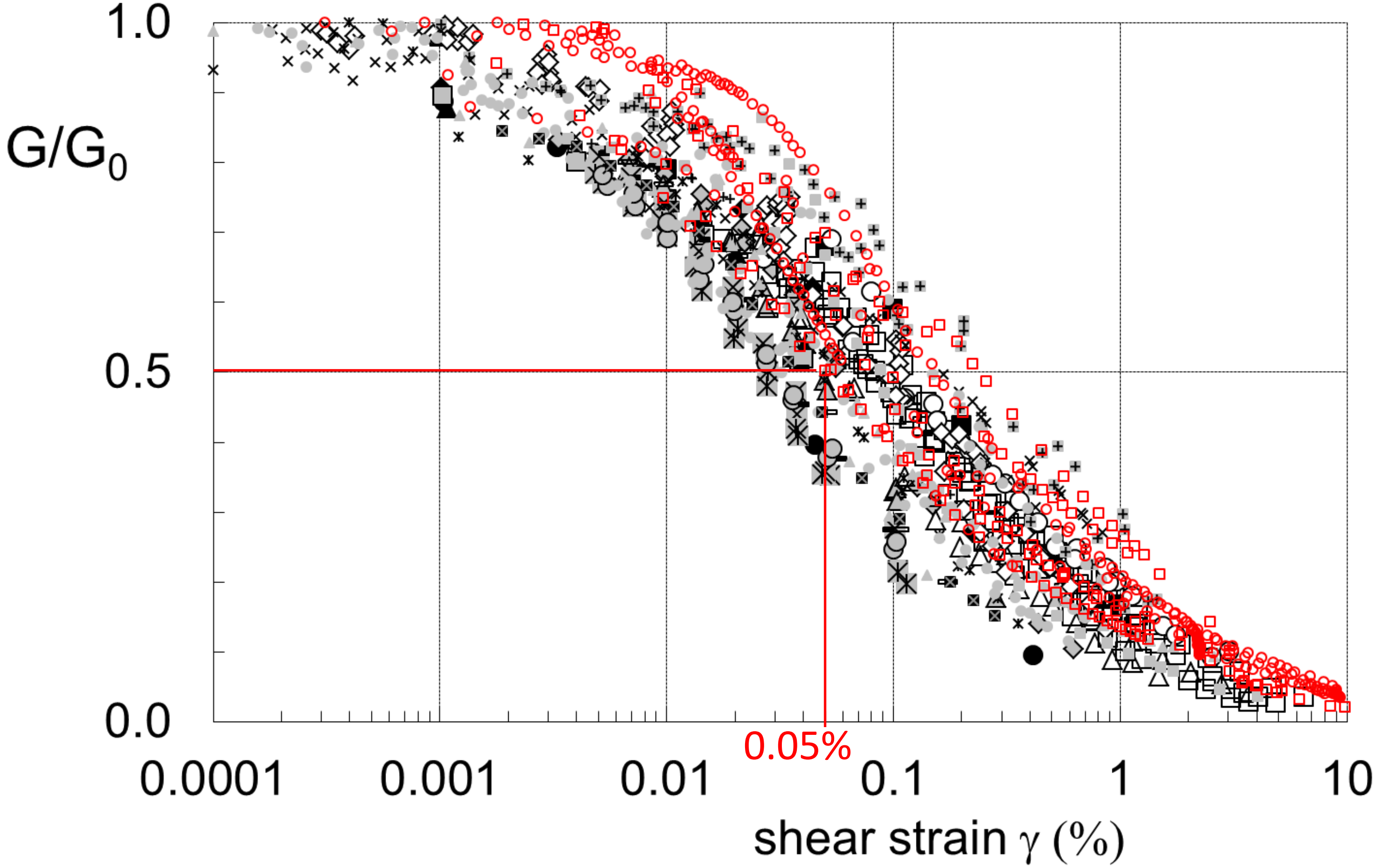
Stiffness: strain-softening #2

- Evolution of a strong contact-force network.
- At random, certain groups of neighbouring soil grains happen to have co-linear contact forces, roughly perpendicular to their contact planes so that they do not slide, and roughly aligned with what is emerging as the principal compressive stress direction.
- These strong chains are progressively “discovered” and relied upon as weaker parts of the contact network slide. Eventually, even the strong chains break under strain as peak strength is reached.

Data of secant G/G_0 for 15 sands



Data of secant G/G_0 for 20 clays



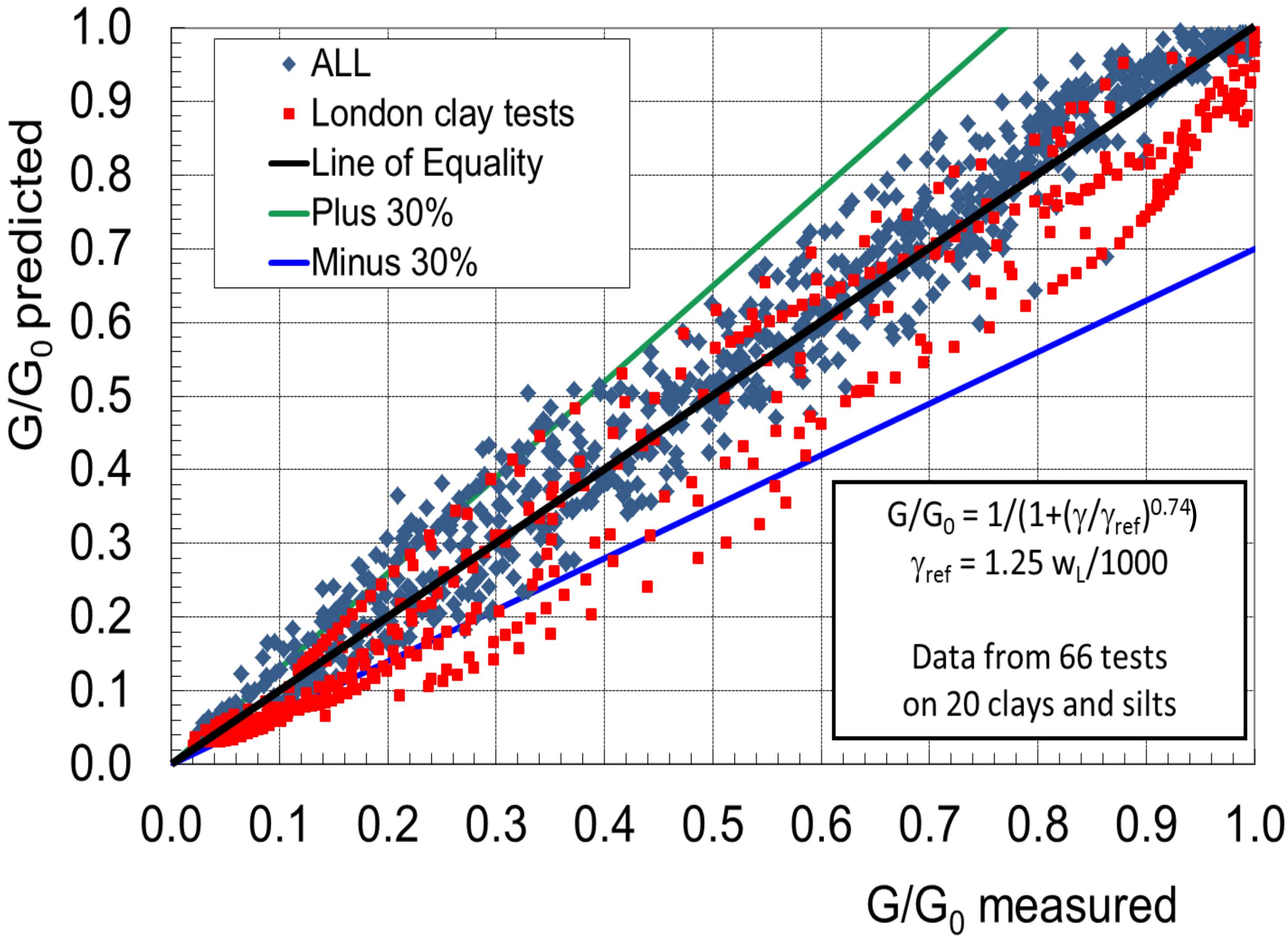
Stiffness: strain-softening #3

- $\frac{G}{G_0} = \frac{1}{1+\left(\frac{\gamma}{\gamma_r}\right)^a}$ so $\tau = \frac{G_0\gamma}{1+\left(\frac{\gamma}{\gamma_r}\right)^a}$ which is "hyperbolic"

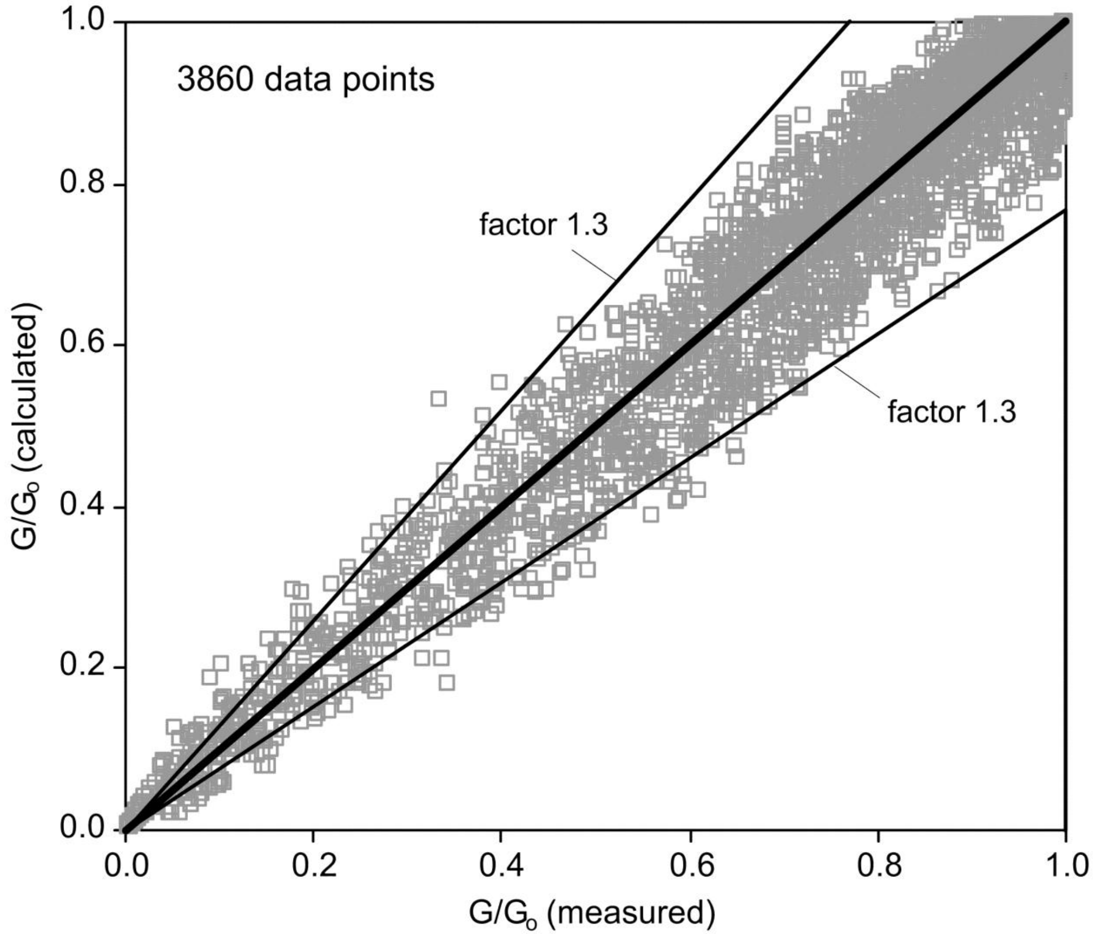
  and note $G/G_0 = 0.5$ at $\gamma = \gamma_r$

- Sands: Oztoprak & Bolton (2012) Geotechnique
  - curvature parameter $a = U_C^{-0.075}$
  - reference strain $\gamma_r = 8\, e\, I_D 10^{-4} + U_C^{-0.3}\, p'\, 10^{-6}$
- Clays: Vardanega & Bolton (2011) IS-Seoul
  - curvature parameter $a = 0.74$
  - reference strain $\gamma_r = 1.25\, w_L\, 10^{-4}$

Reliability of G/G_0 prediction for clay



Reliability of G/G_0 prediction for sand



Overview: non-linear stiffness at $\gamma < 1\%$

- Stiffness at very small strains: using B to correct G_0
  - Measure G_0 and then deduce B to correct future G_0 for p'.
  - Effect of anisotropy: choosing a pertinent test
- Secant G/G_0 can be estimated ± 30% (2 st. dev.)
  - Due to contact sliding required to evolve a strong network
  - Quasi-hyperbolic $\tau - \gamma$ curves for $\gamma < 1\%$, with $G/G_0 = 0.5$ at $\gamma = \gamma_r$
  - Reference strain γ_r is the salient additional parameter
  - In clay γ_r depends on w_L: Vardanega & Bolton (2011)
  - In sand γ_r depends on U_C, I_D, p' : Oztoprak & Bolton (2012)
- So engineers should infer G_0, estimate γ_r and use existing non-linear settlement calculations.

Conclusion

- Granular simulations plus laboratory databases can be used to clarify micro-mechanisms and propose a few soil parameters (B and γ_r for stiffness, σ_c for clastic yielding, ϕ_{crit} for strength) that capture the essentials of non-linear soil behaviour using a few dimensionless groups.
- Statistical analysis is then possible, using coefficients of variation derived from databases, so that predictions both of serviceability and safety can be made in a reliability framework without using safety factors.
- But we need to check that the characterization of a project soil fits within an existing database, and always look for laboratory evidence of new micro-mechanisms.

Collaborators

- I have enjoyed working on granular mechanics with outstanding PhD students including Damon Lee (1992), Glenn McDowell (1996), Doug Robertson (2000), Dave White (2002), Helen Cheng (2004), George Marketos (2008), & Fiona Kwok (2008), and with Yukio Nakata when he came here as a visitor.
- I was also lucky to work with Paul Vardanega (2012), and with Sadik Oztoprak visiting from Istanbul, who put a lot of effort into the statistical analysis of soil test databases.
- And I had the very good fortune to be introduced to Masayuki Hyodo in the 1990s, and later Mingjing Jiang, who hosted the previous Symposia of what is now TC105, introducing us all to an even richer variety of micro-macro thinking.

Particular thanks

- Of course, I have been equally lucky to have colleagues such as Kenichi Soga and Matthew Kuo conducting exciting research on soil behaviour here in Cambridge.
- And I am particularly grateful to Kenichi for suggesting that we have this conference here, and then for organizing it!
- Finally, I am delighted that some of my friends agreed to make the following presentations in this session, and that you are here to listen to us!