To see and compute as never before in granular materials: the avatar concept

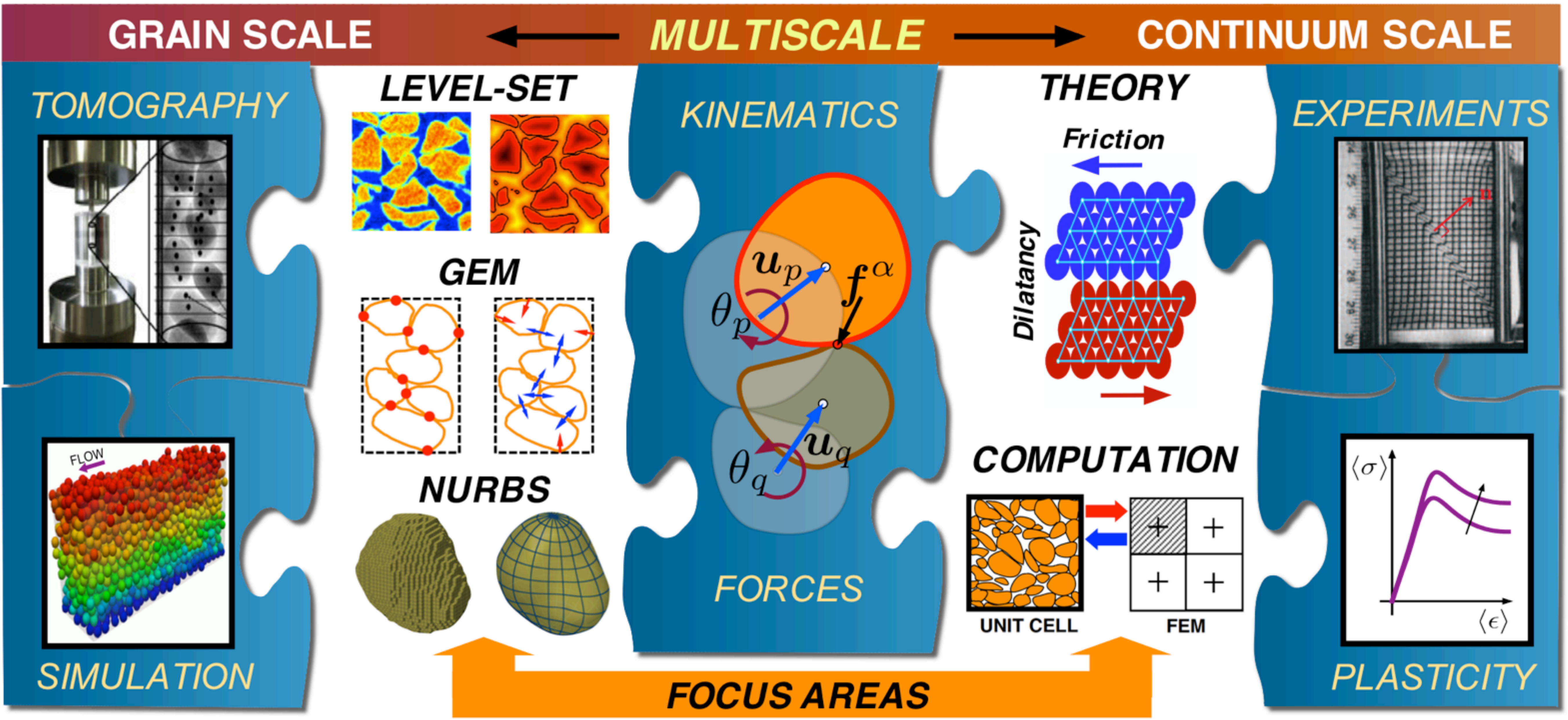
José E. Andrade
Mechanical & Civil Engineering

Keynote IS-Cambridge, UK, September 2014

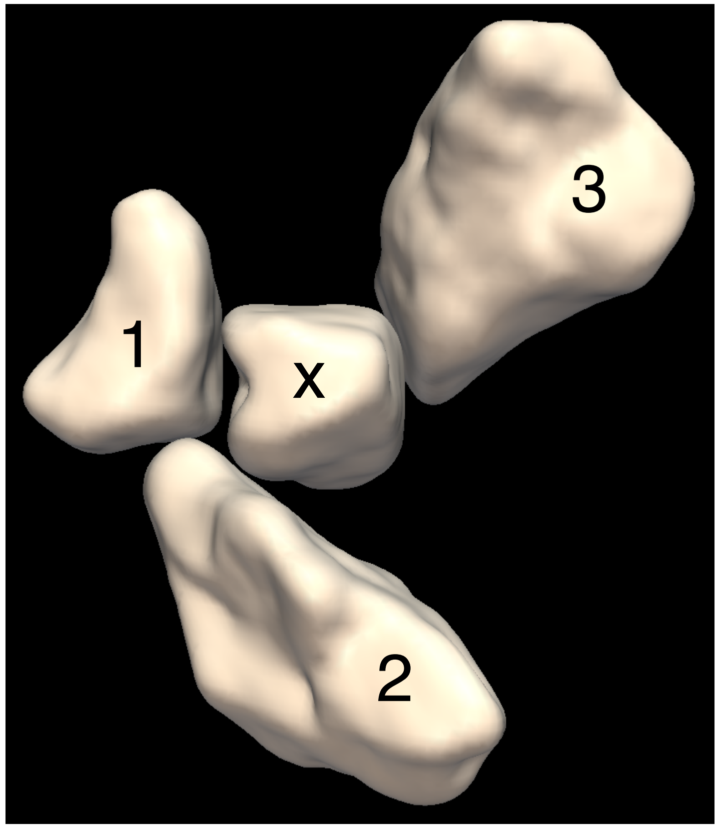
THOR

NSF

Keck
INSTITUTE FOR SPACE STUDIES



GRAIN SCALE
MULTISCALE
CONTINUUM SCALE
TOMOGRAPHY
LEVEL-SET
KINEMATICS
THEORY
EXPERIMENTS
Friction
Dilatancy
GEM
FLOW
NURBS
COMPUTATION
UNIT CELL
FEM
FORCES
SIMULATION
FOCUS AREAS
PLASTICITY
$\langle\sigma\rangle$
$\langle\epsilon\rangle$
u_p
θ_p
f^α
u_q
θ_q



Multiscale

Grain Scale (~mm)

Newtonian Mechanics

Continuum Scale (>m)

Continuum Mechanics

$$F = ma$$

discrete

concept: force F

$$\nabla \cdot \sigma = \rho a$$

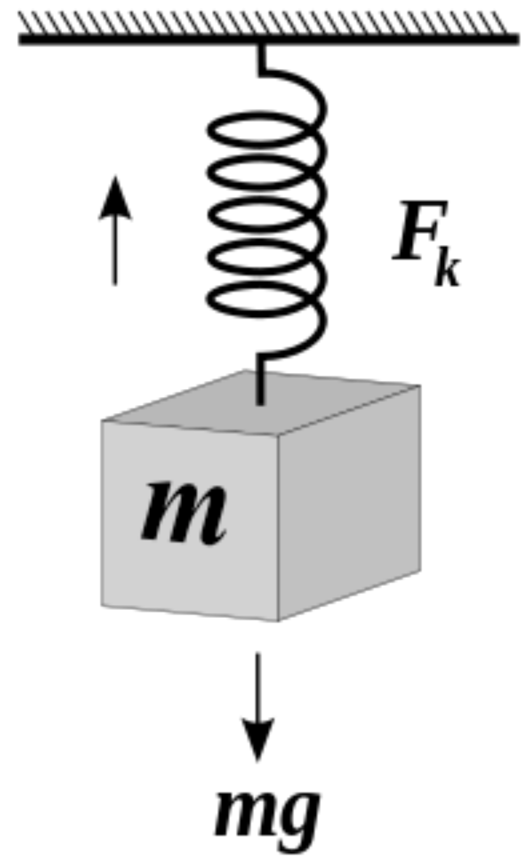
average, bulk

concept: stress σ

Concept of Force

Isaac Newton: 1643-1727

Robert Hooke: 1635-1703



$$F = ma$$

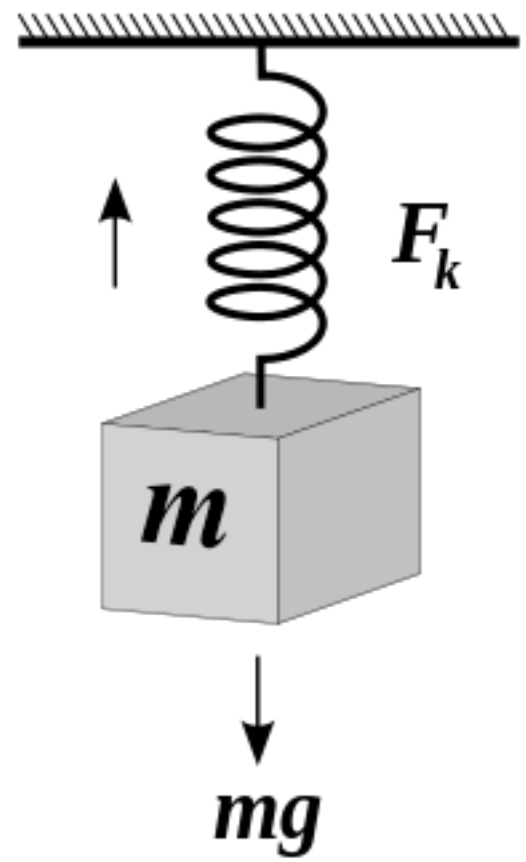
$$F = k\delta$$

Concept of Force

Isaac Newton: 1643-1727

$$F = ma$$

force relates to momentum



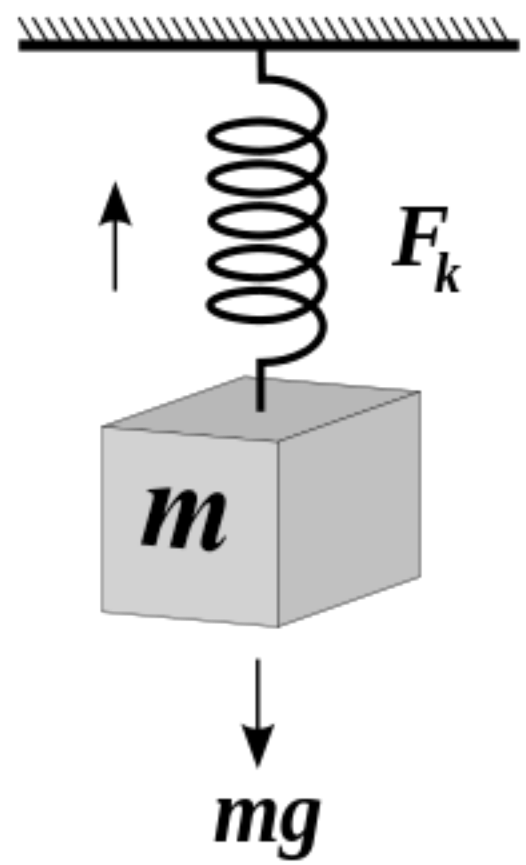
Robert Hooke: 1635-1703

$$F = k\delta$$

Concept of Force

Isaac Newton: 1643-1727

Robert Hooke: 1635-1703



$$F = ma$$

force relates to momentum

$$F = k\delta$$

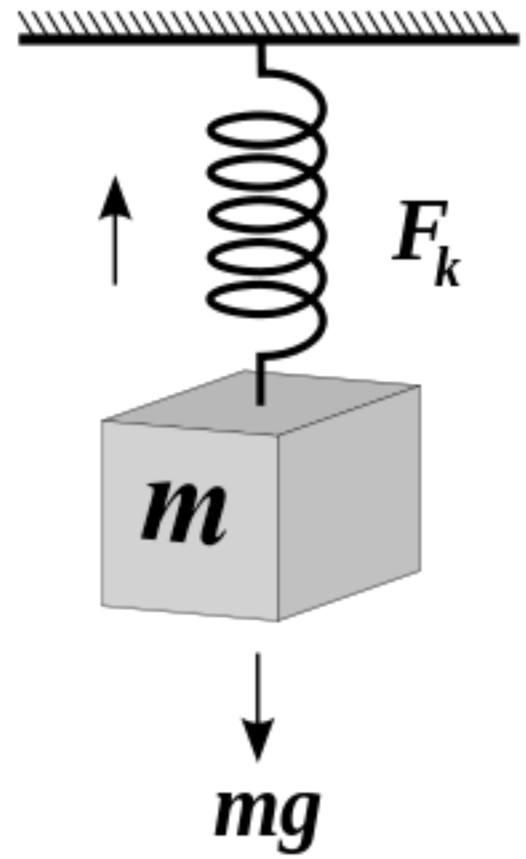
ceiiinosssttuv

Concept of Force

Isaac Newton: 1643-1727

$$F = ma$$

force relates to momentum



Robert Hooke: 1635-1703

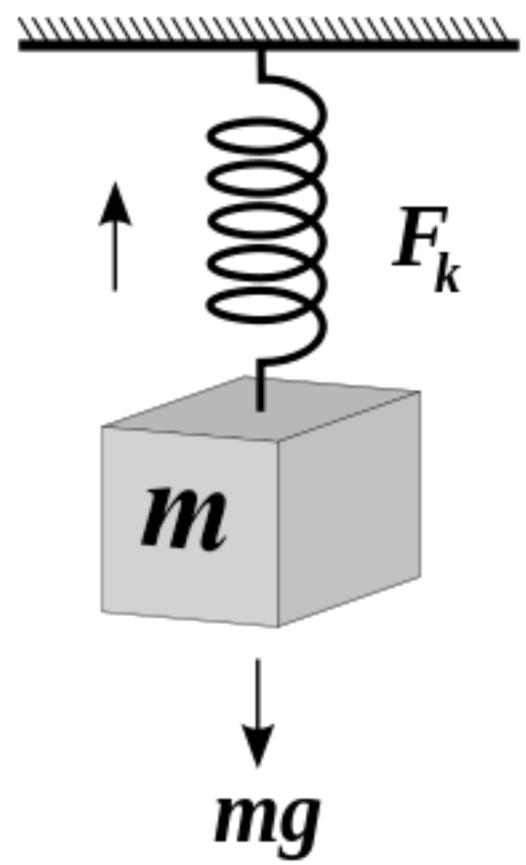
$$F = k\delta$$

Concept of Force

Isaac Newton: 1643-1727

$$F = ma$$

force relates to momentum



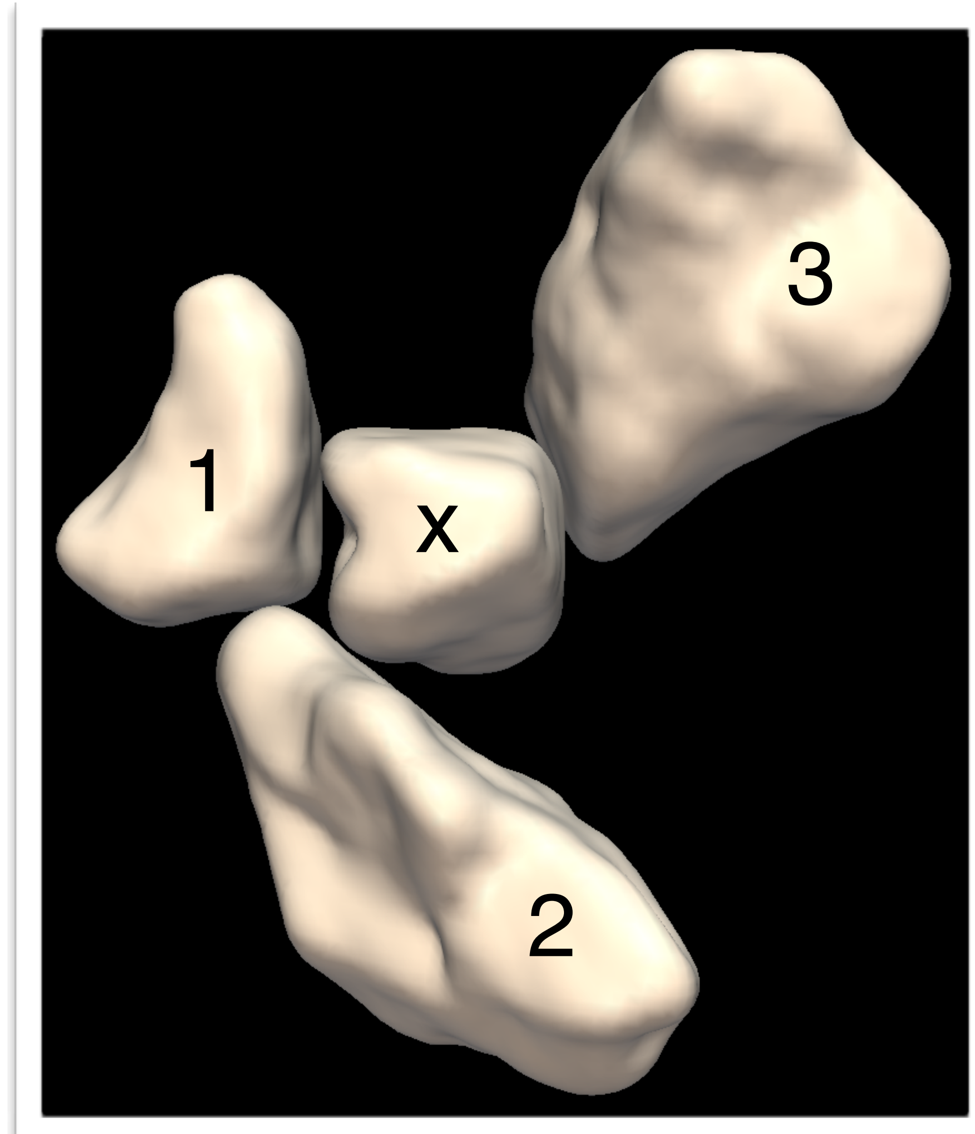
Robert Hooke: 1635-1703

$$F = k\delta$$

ut tensio sic vis

physical features

behavior

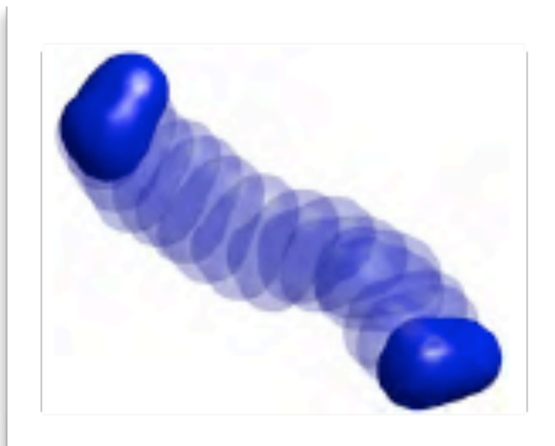


morphology = shape

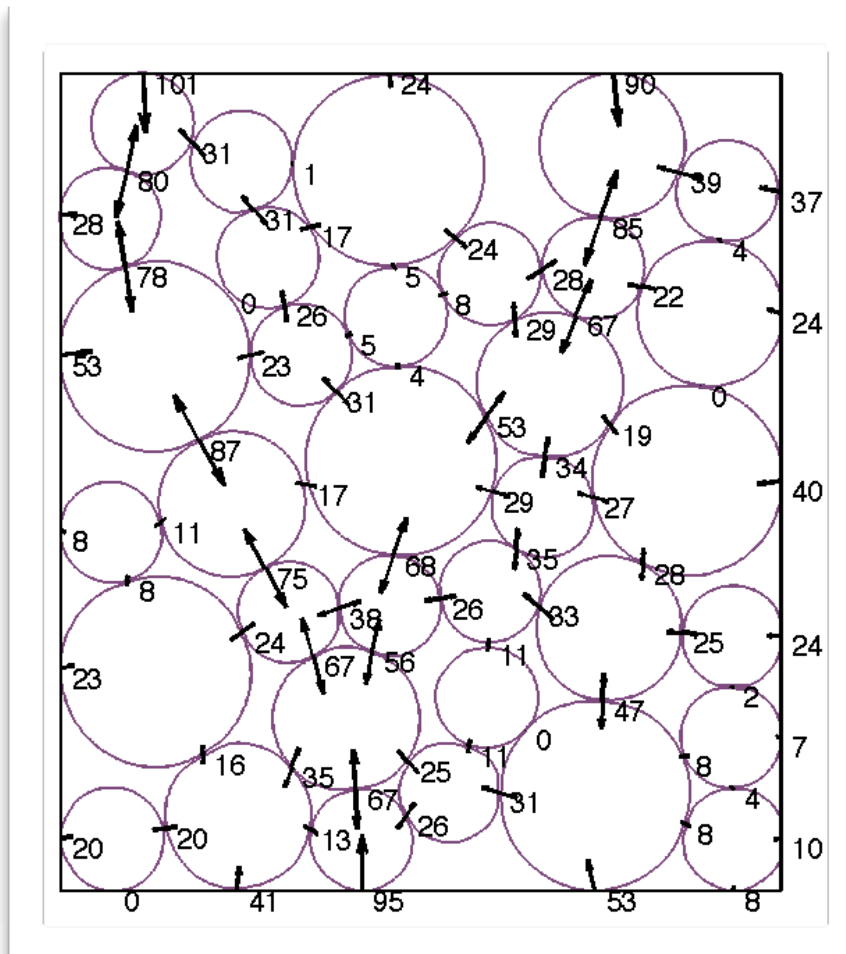
behavior = kinematics & forces

The avatar concept

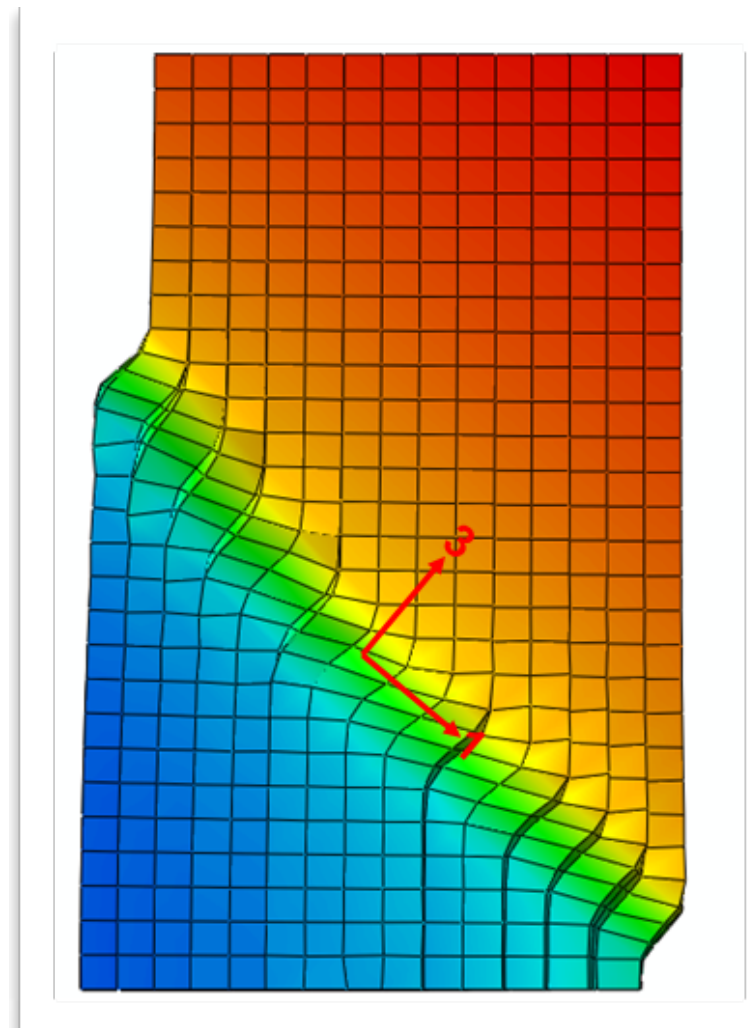
Today's menu: Avatar



measuring kinematics

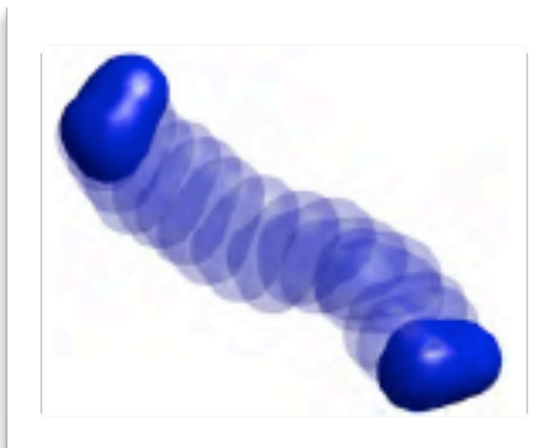


measuring forces

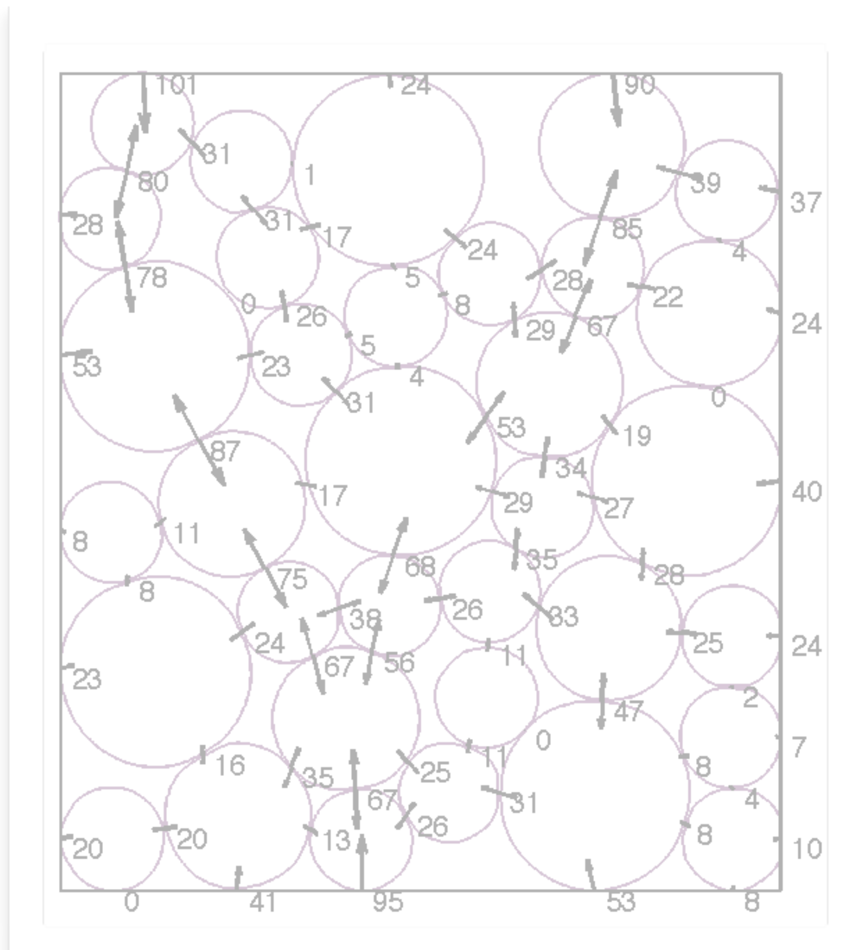


predicting behavior

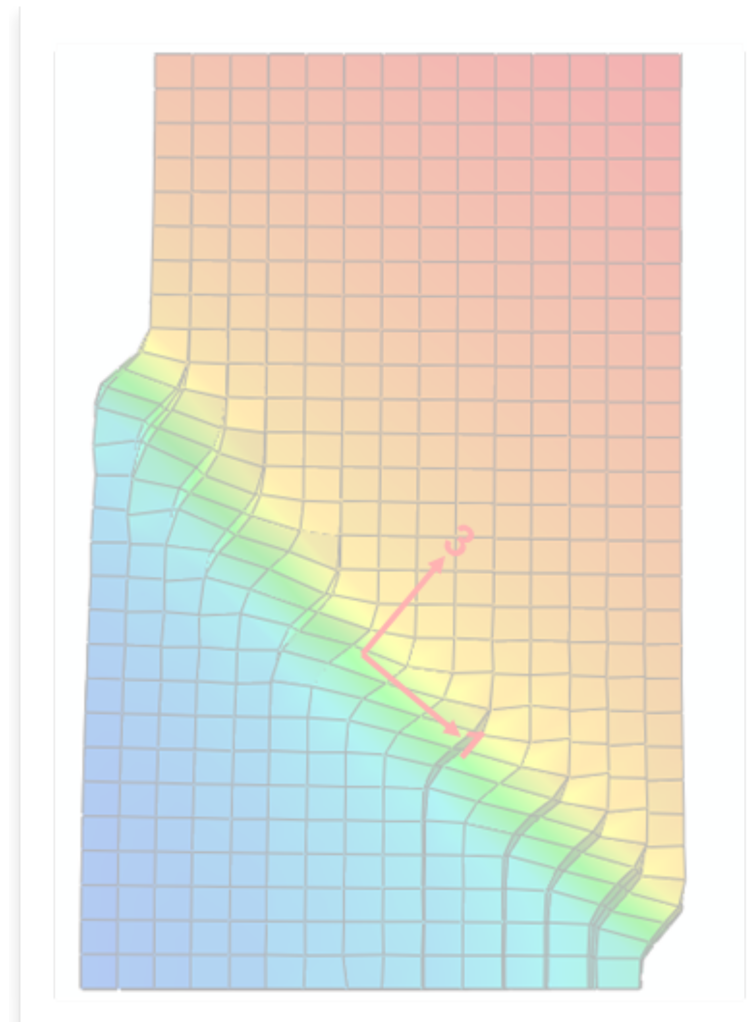
Today's menu: Avatar



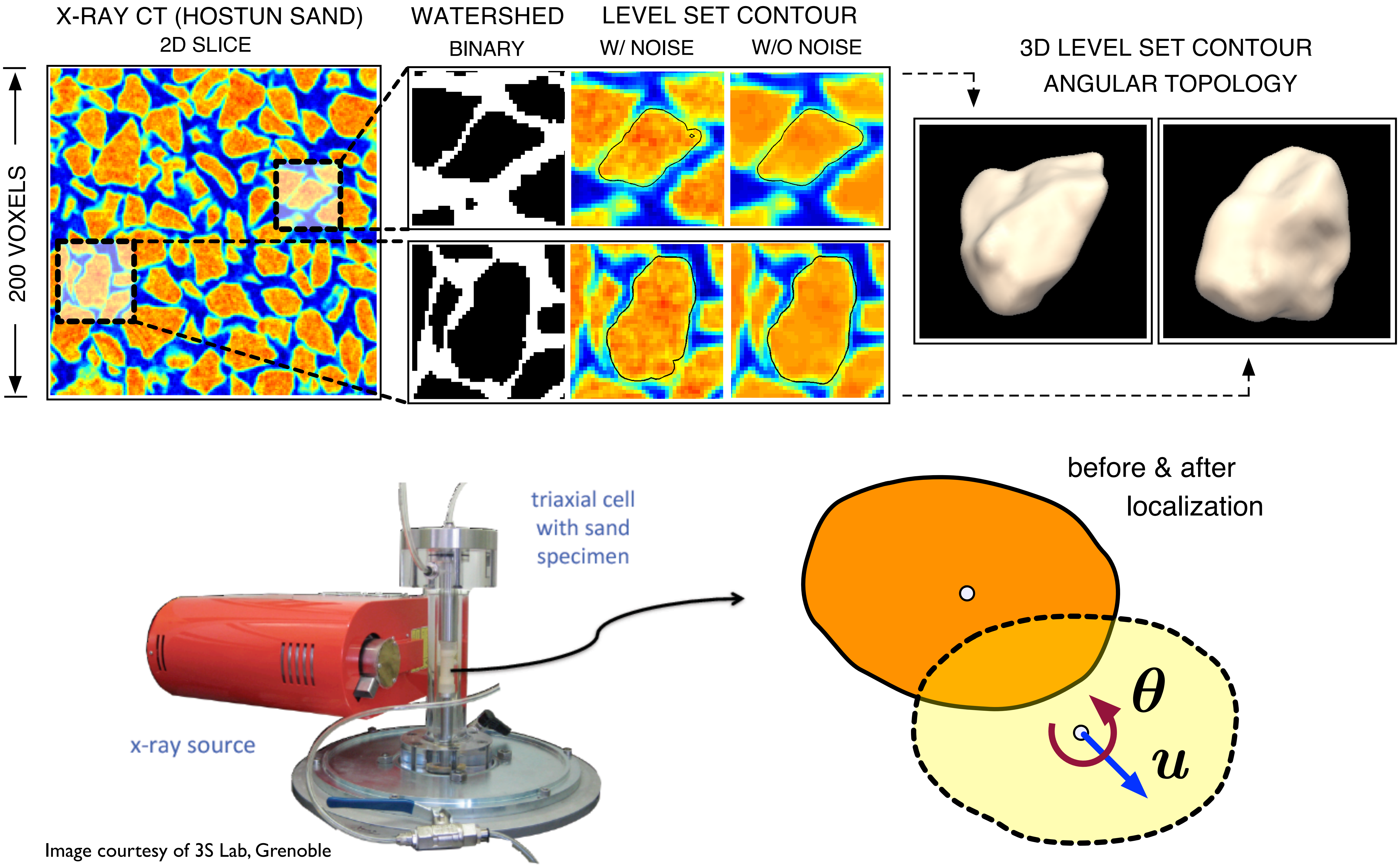
measuring kinematics

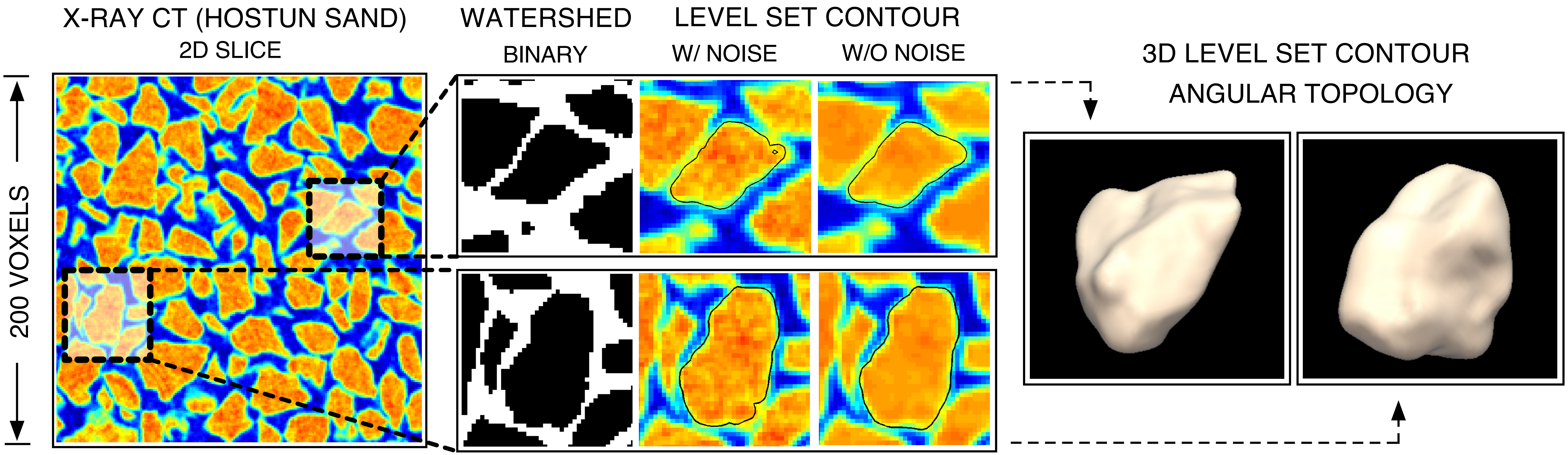


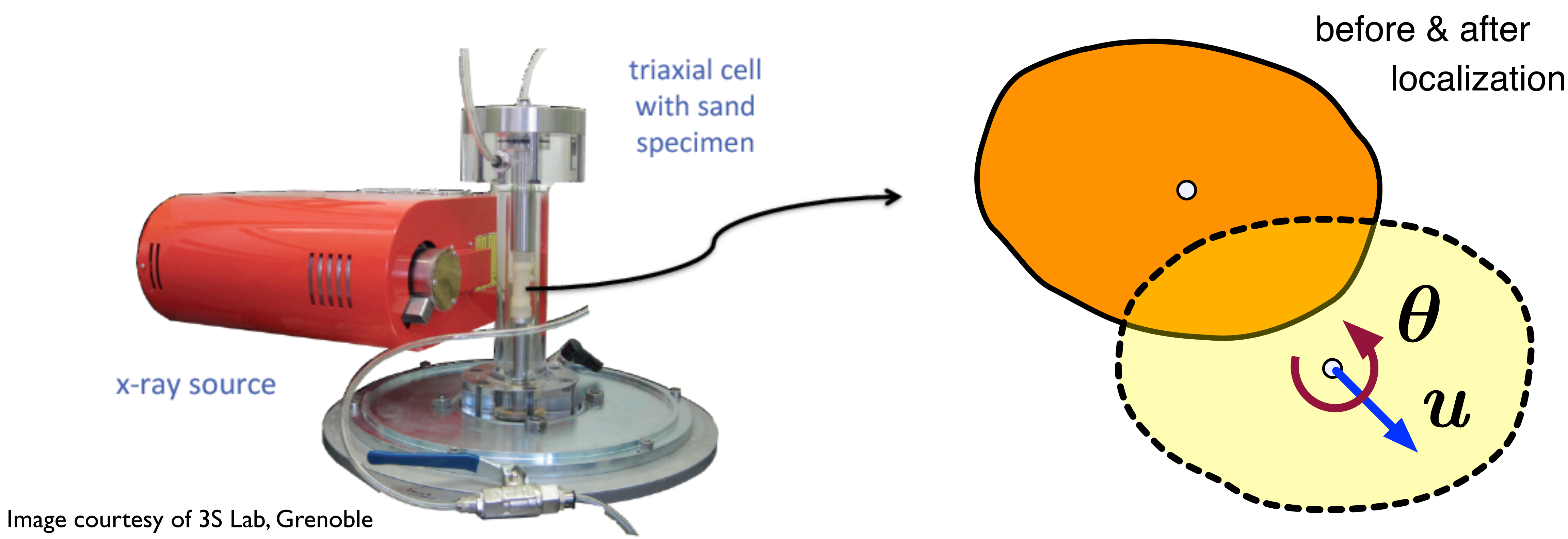
measuring forces



predicting behavior

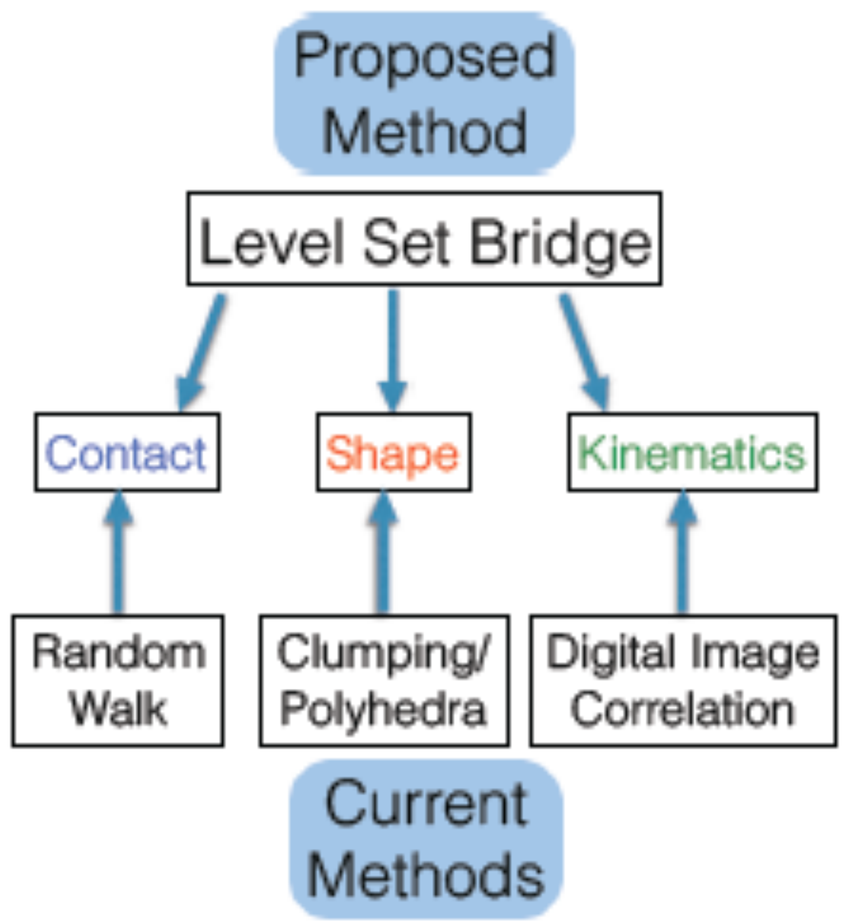

X-RAY CT (HOSTUN SAND)
2D SLICE
200 VOXELS
WATERSHED
BINARY
LEVEL SET CONTOUR
W/ NOISE
W/O NOISE
3D LEVEL SET CONTOUR
ANGULAR TOPOLOGY
triaxial cell
with sand
specimen
x-ray source
Image courtesy of 3S Lab, Grenoble
before & after
localization
θ
u



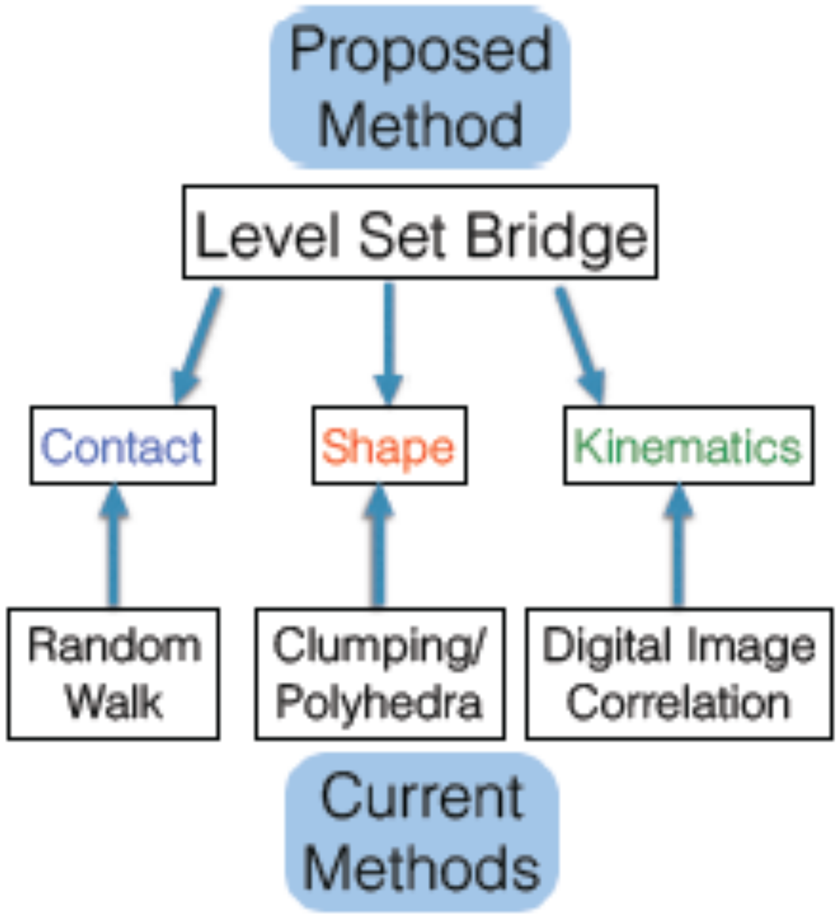


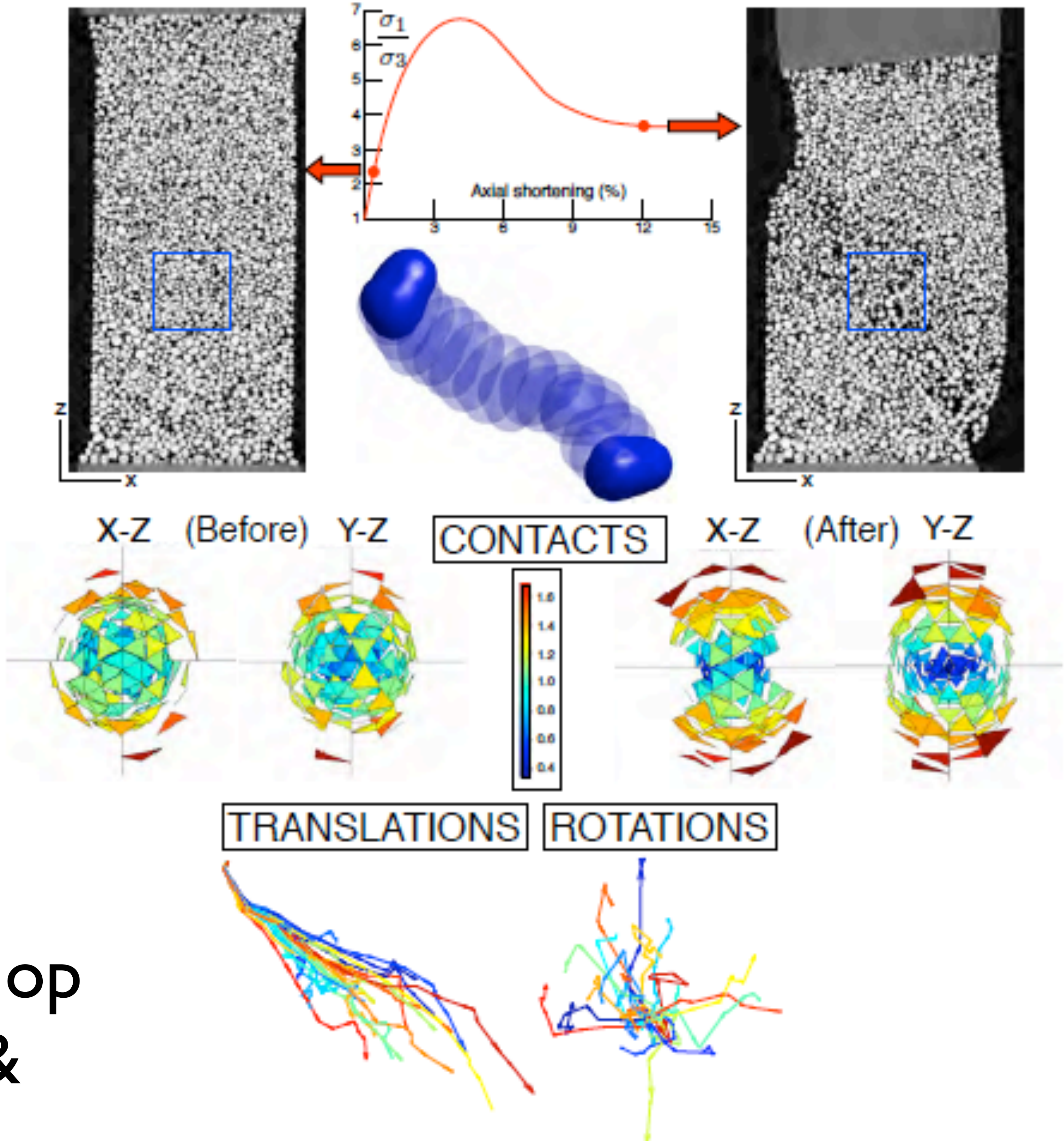
experimentally capture morphology & kinematics

Level-set: a one-stop shop for characterization & computations



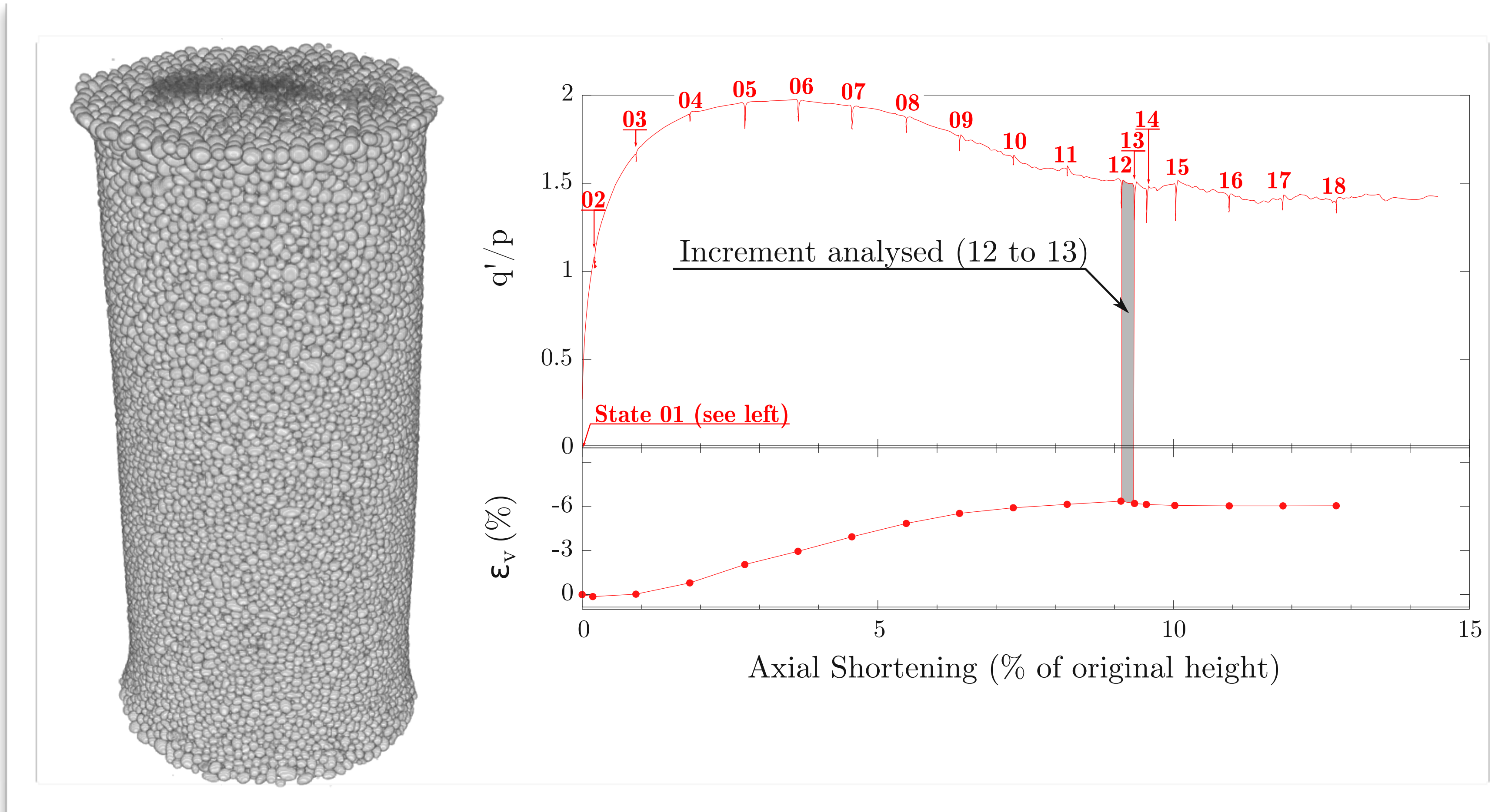
Level-set: a one-stop shop for characterization & computations

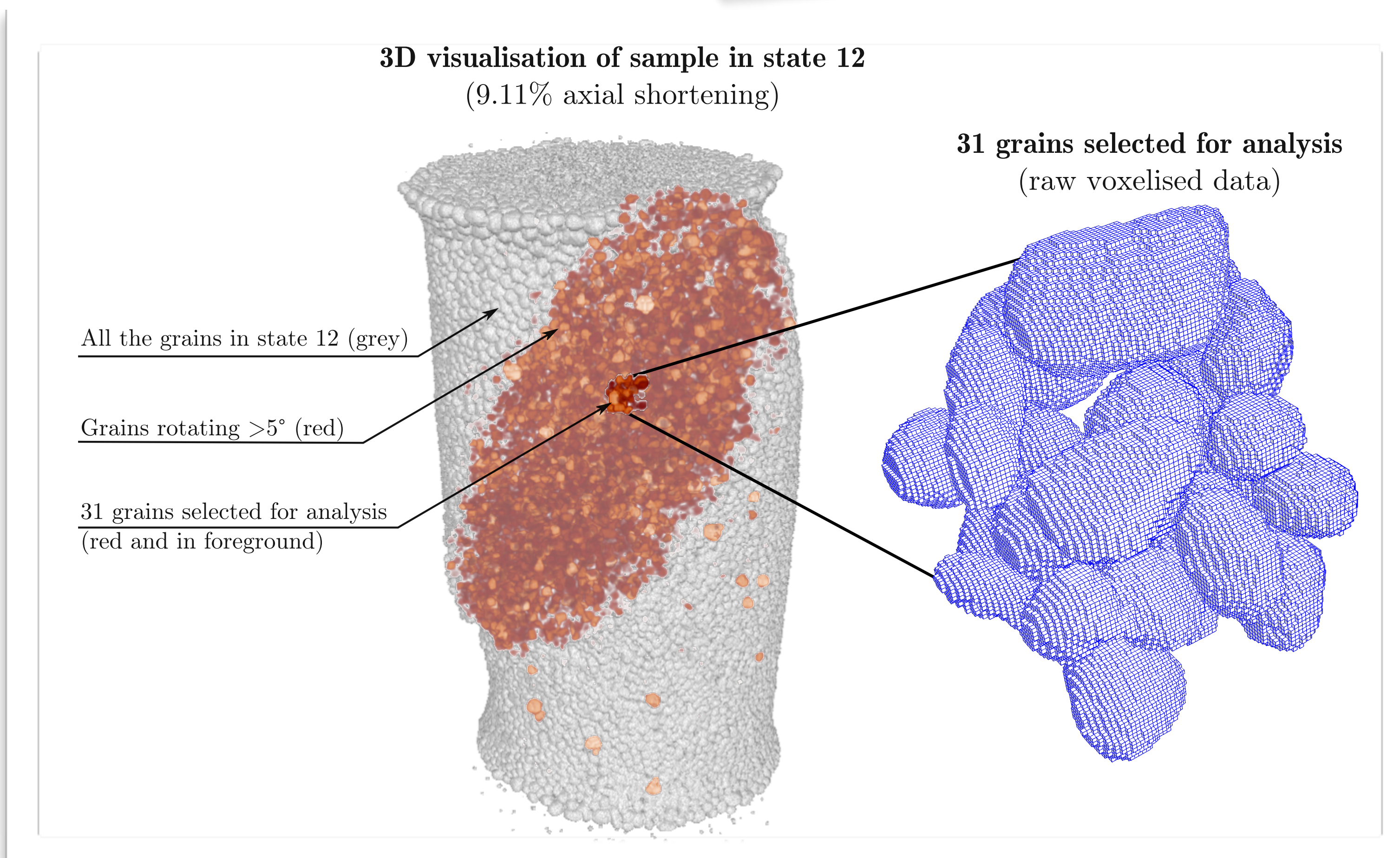




Level-set: a one-stop shop for characterization & computations

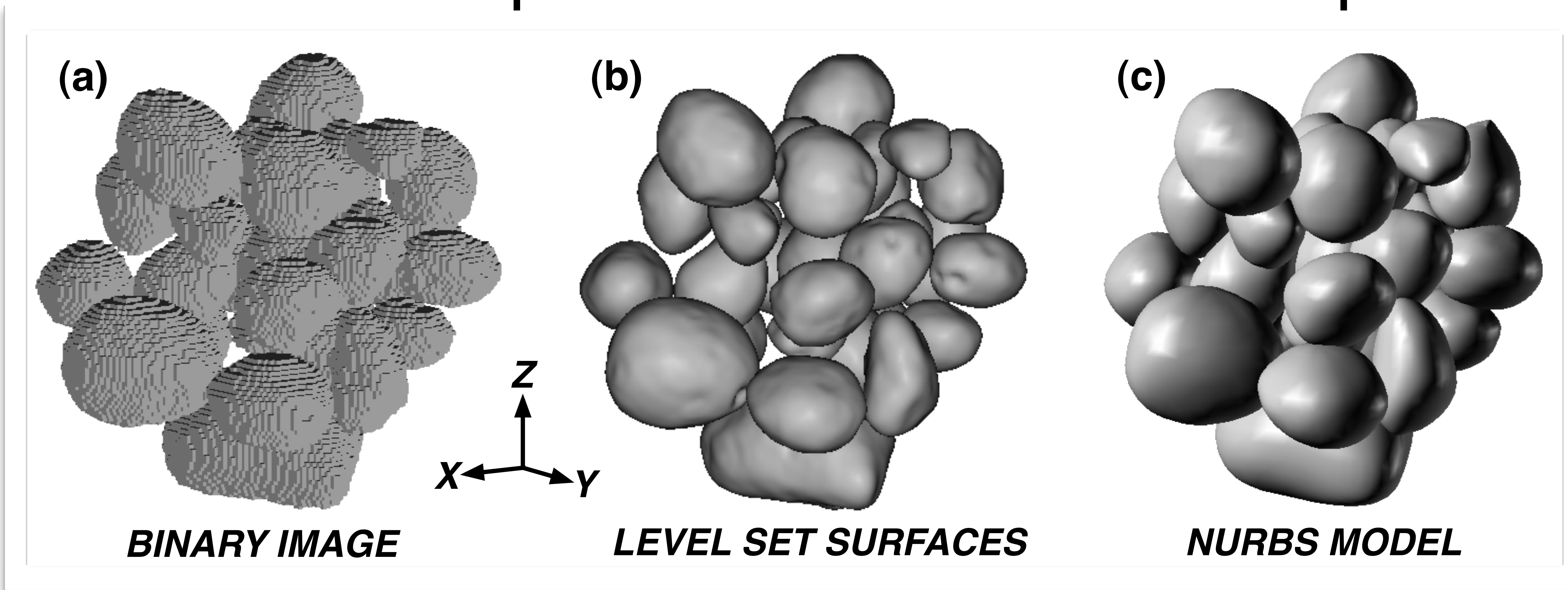
Macro-micro experimental data

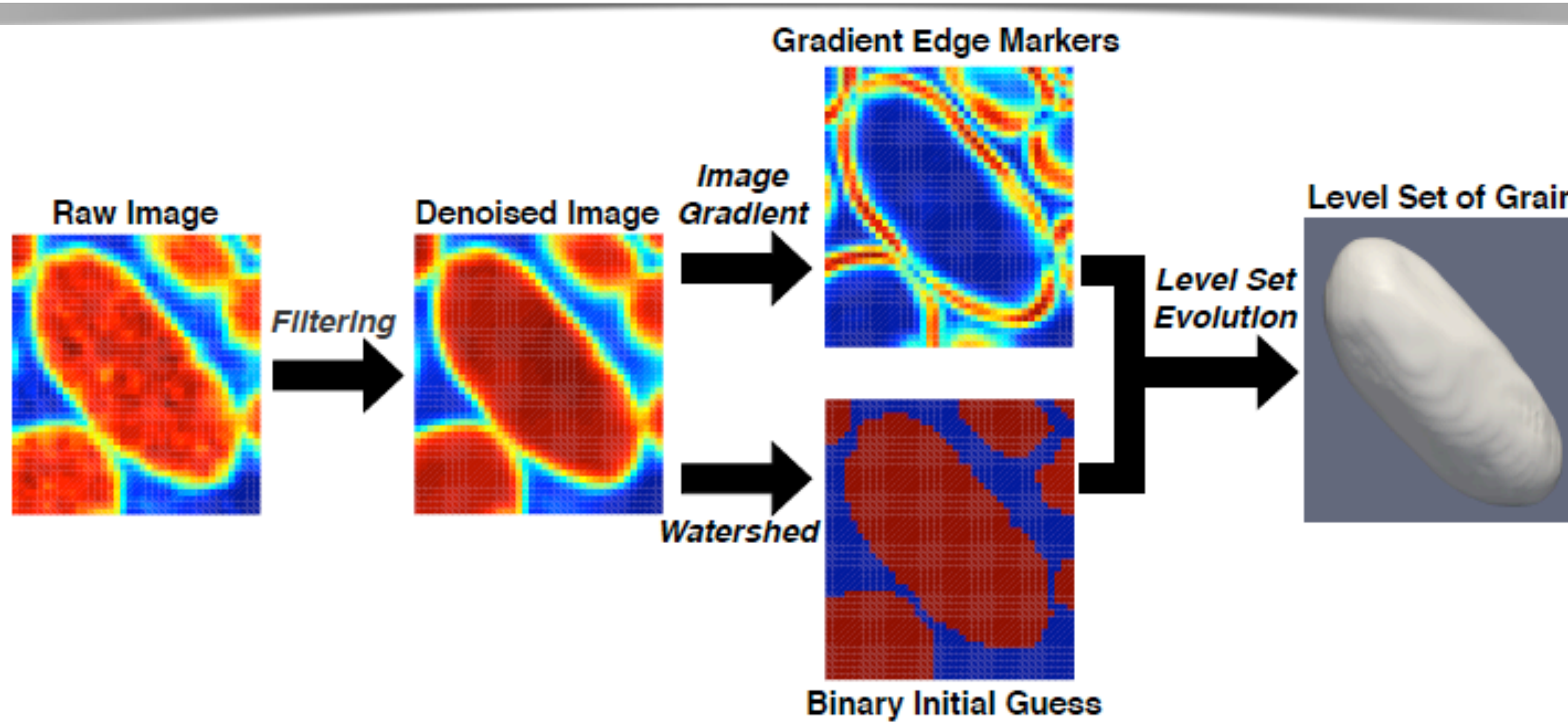




Macro-micro experimental data @ failure

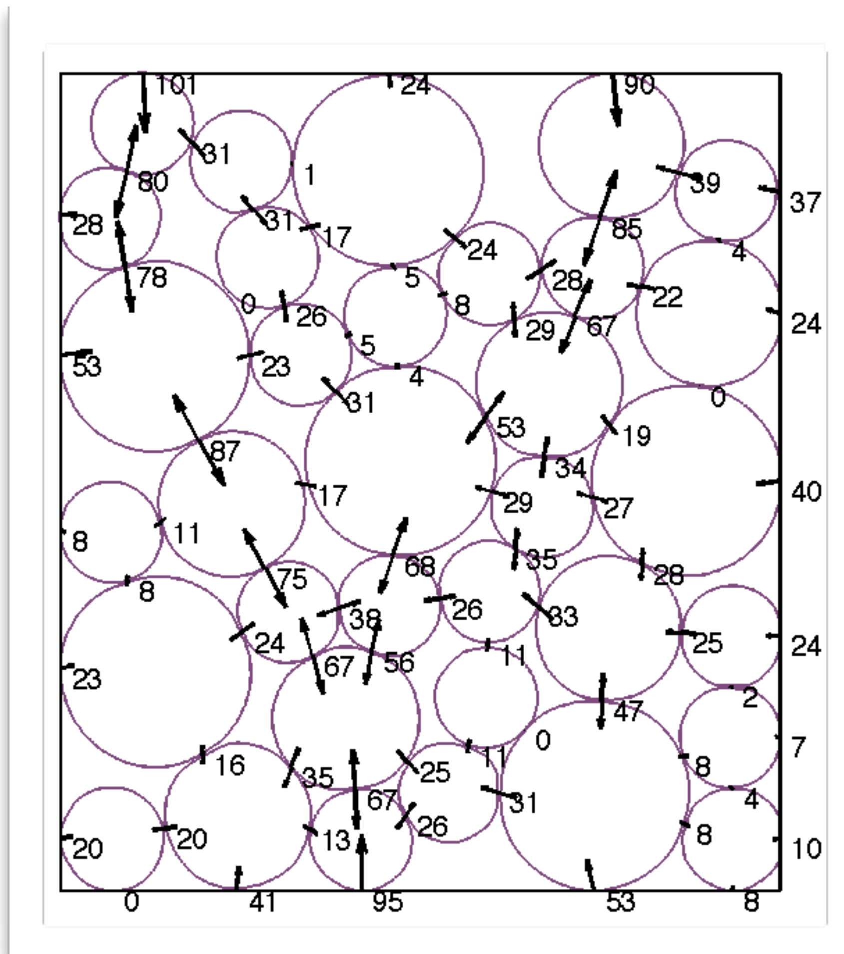
Construct computational model of real 3D particles



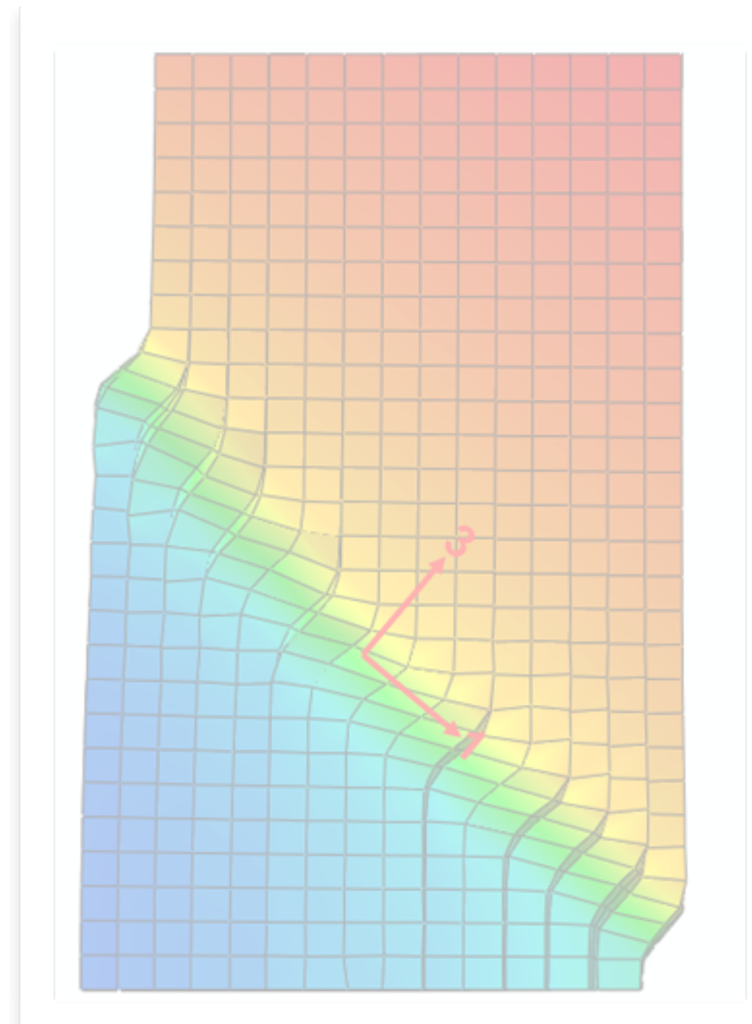


Today's menu: Avatar

measuring kinematics



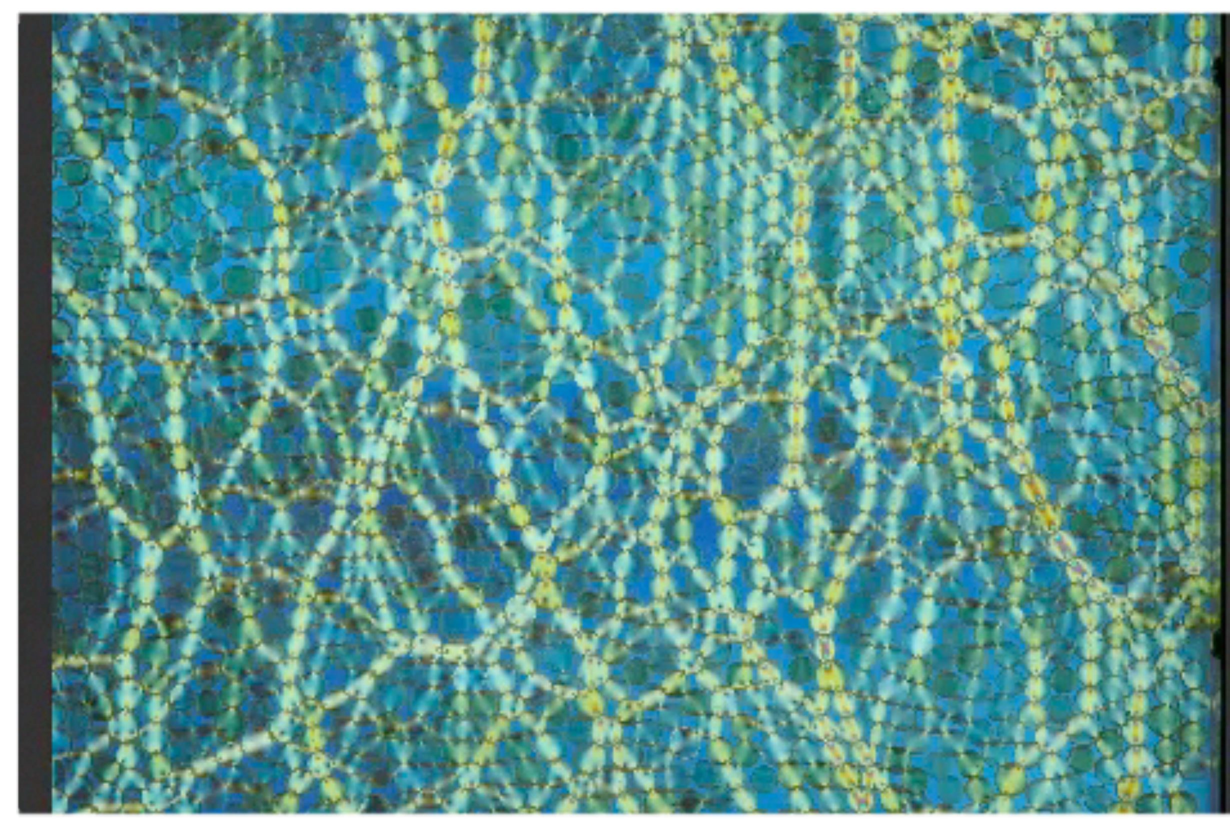
measuring forces



predicting behavior

Current workhorse: Photoelasticity

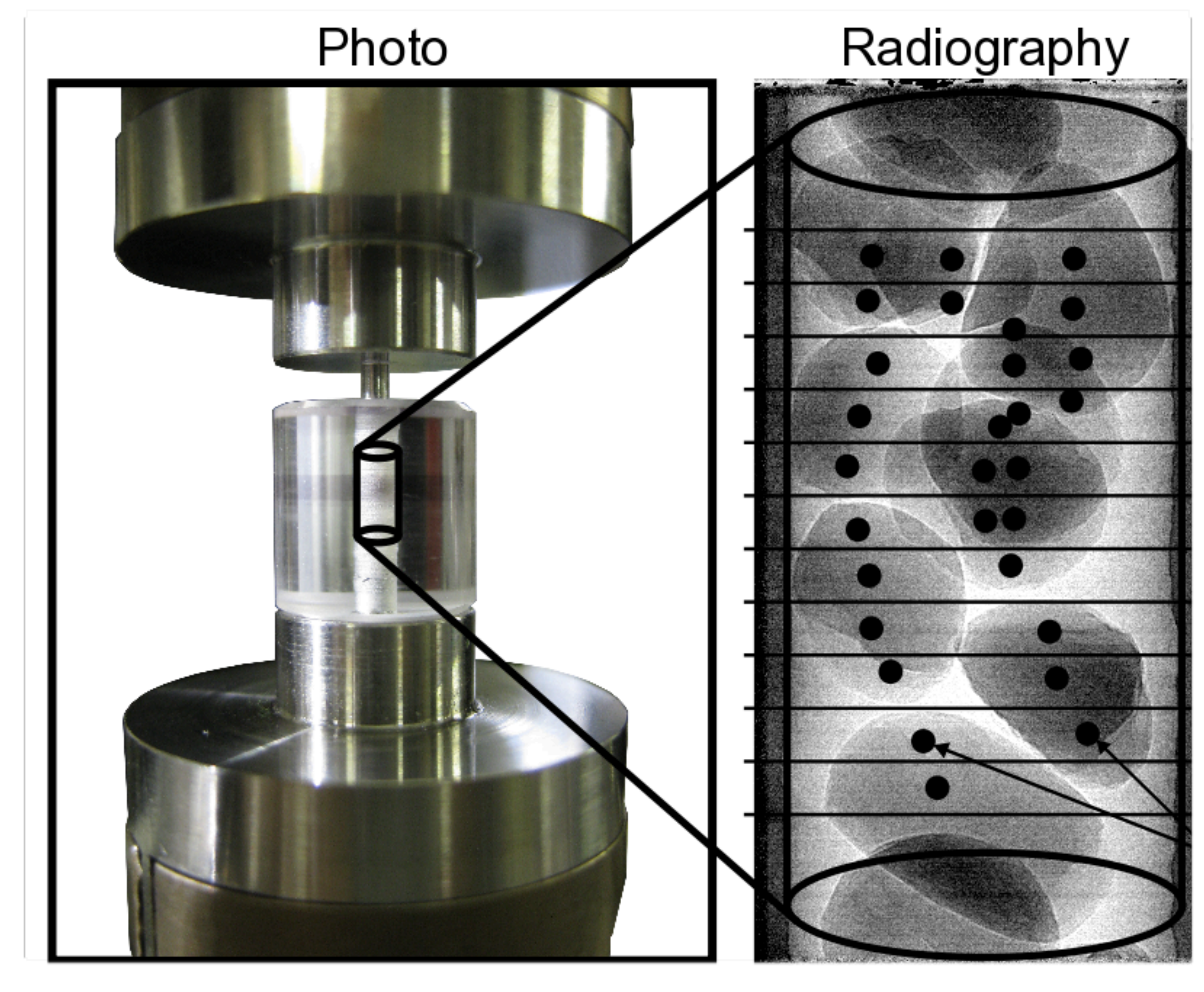
- Great in 2D
- Great insight into force-chain evolution
- Cannot provide accurate quantitative values of contact force
- Cannot work in opaque materials (e.g., sand)

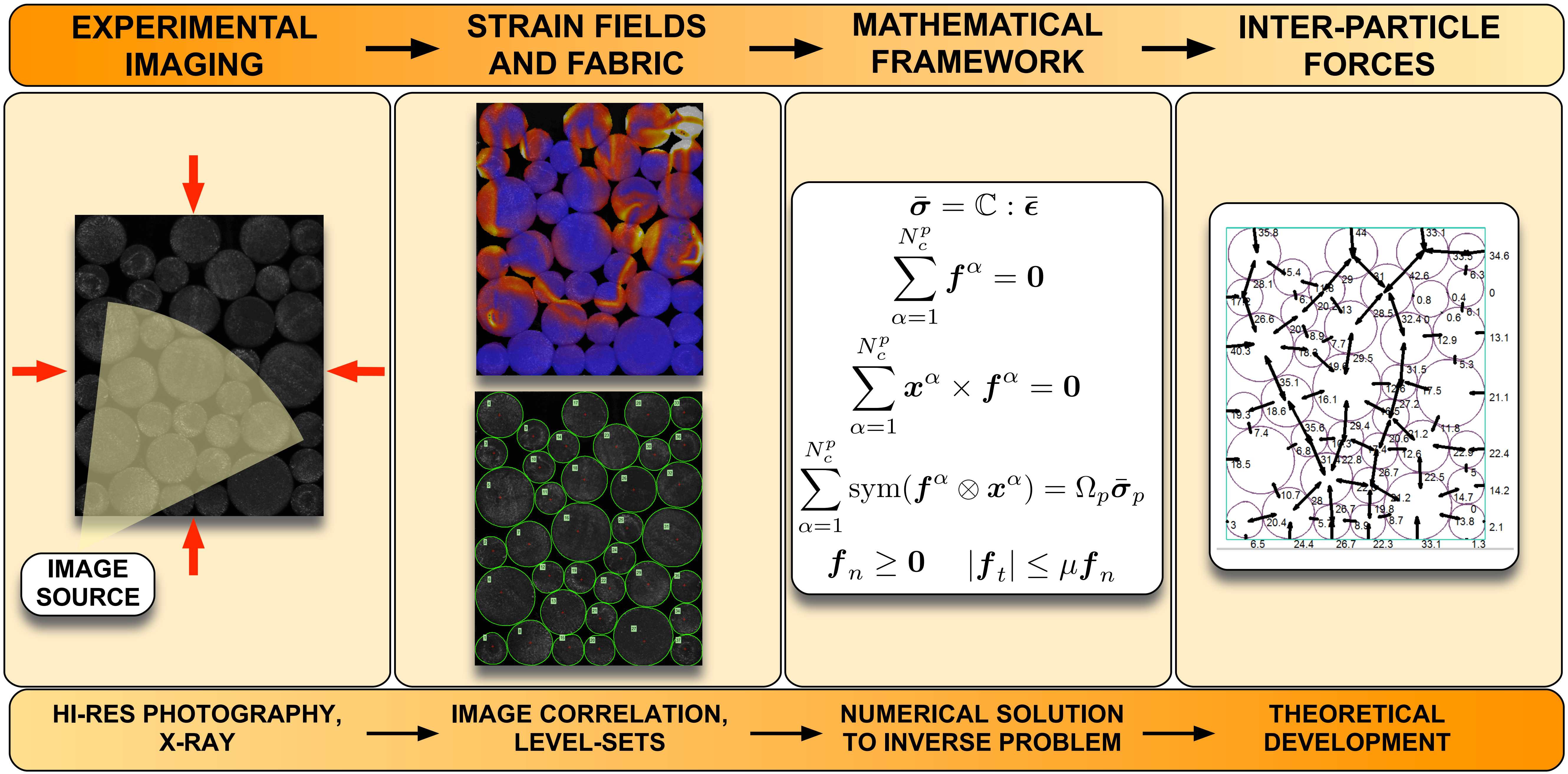


Source- Jie Zhang/Duke University

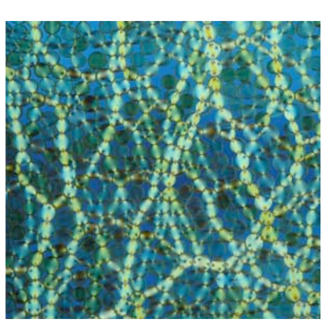
Can contact forces be measured?

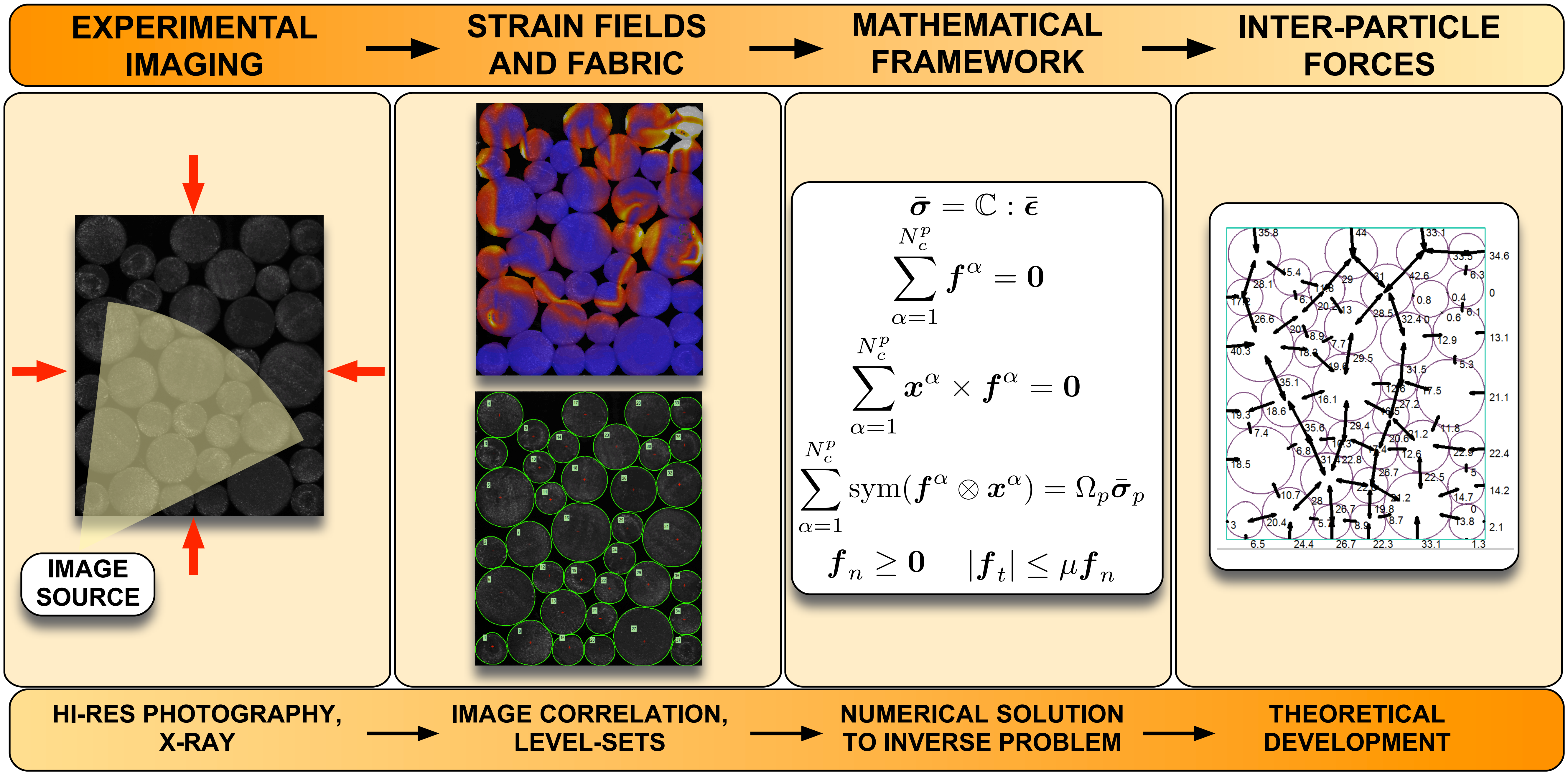
- 3DXRCT & 3DXRD: grain topology, kinematics & average grain strains
- Fundamental question: how to use information (constitutive modeling)?
- Missing link: grain contact forces Vs. stress

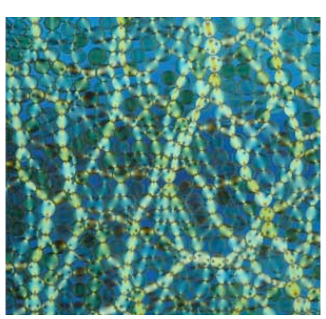




EXPERIMENTAL IMAGING
STRAIN FIELDS AND FABRIC
MATHEMATICAL FRAMEWORK
INTER-PARTICLE FORCES
IMAGE SOURCE
$\bar{\boldsymbol{\sigma}} = \mathbb{C} : \bar{\boldsymbol{\epsilon}}$
$\sum_{\alpha=1}^{N_c^p} \boldsymbol{f}^{\alpha} = \mathbf{0}$
$\sum_{\alpha=1}^{N_c^p} \boldsymbol{x}^{\alpha} \times \boldsymbol{f}^{\alpha} = \mathbf{0}$
$\sum_{\alpha=1}^{N_c^p} \mathrm{sym}(\boldsymbol{f}^{\alpha} \otimes \boldsymbol{x}^{\alpha}) = \Omega_p \bar{\boldsymbol{\sigma}}_p$
$\boldsymbol{f}_n \geq \mathbf{0} \quad |\boldsymbol{f}_t| \leq \mu \boldsymbol{f}_n$
HI-RES PHOTOGRAPHY, X-RAY
IMAGE CORRELATION, LEVEL-SETS
NUMERICAL SOLUTION TO INVERSE PROBLEM
THEORETICAL DEVELOPMENT





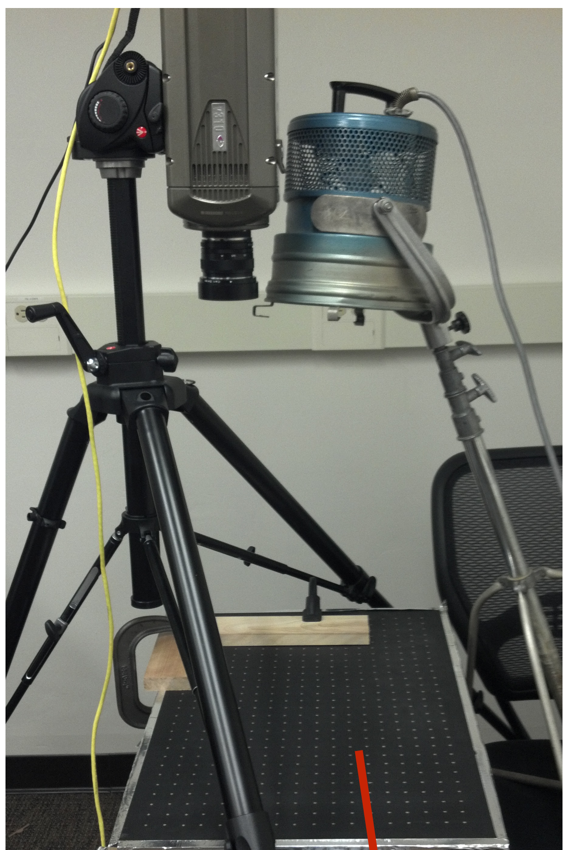


experimentally capture inter-particle forces

Dynamic Validation of Method

Dynamic Validation of Method

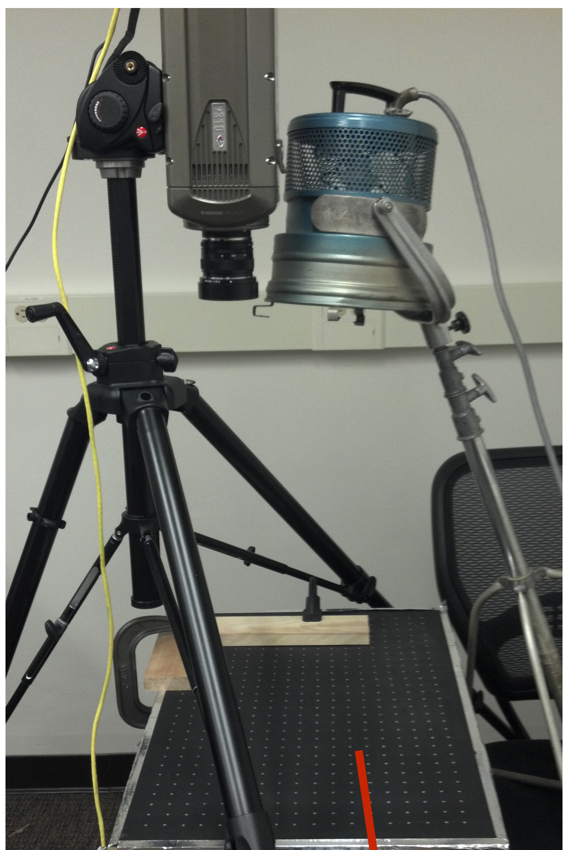
Phantom v310 high-speed camera



Frictionless “air hockey” table
(only forces are inter-particle)

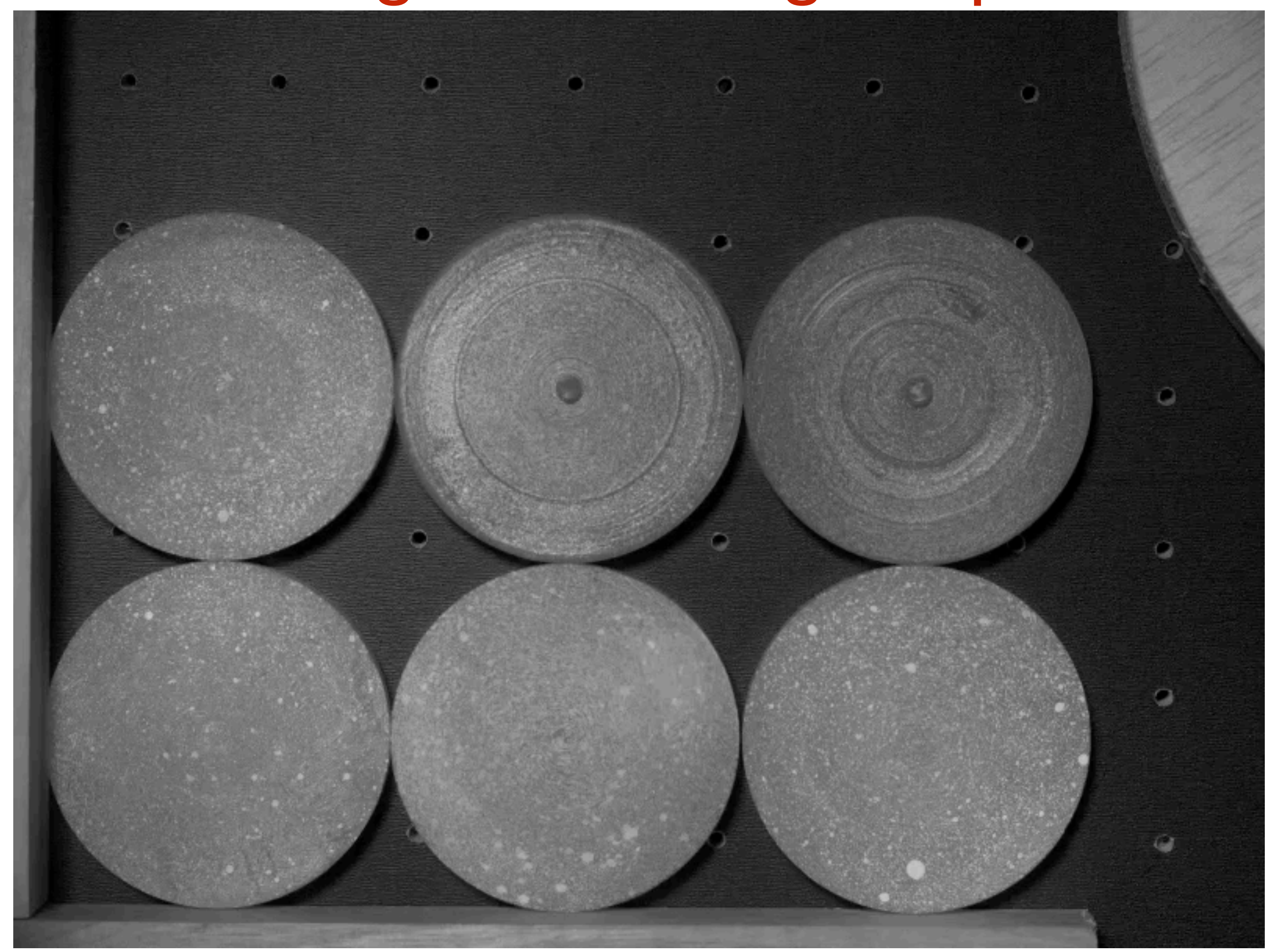
Dynamic Validation of Method

Phantom v310 high-speed camera



Frictionless "air hockey" table
(only forces are inter-particle)

Rubber "grains" and rigid impactor



(1/500 real time)

v_{0x}=-1.14m/s v_{0y}=-0.66m/s

More unknowns than momentum equations

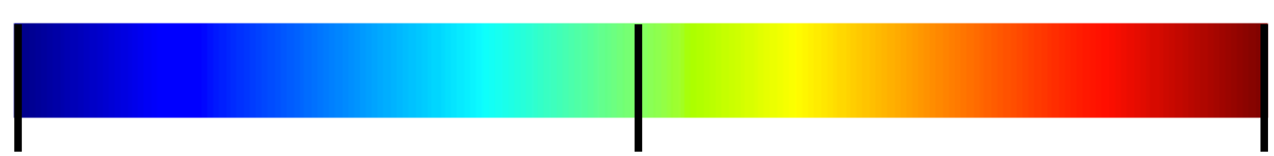
Dynamic Validation of Method

Digital Image Correlation (Vic2d): ϵ_{xy}

FEM Simulation (Abaqus/Explicit): ϵ_{xy}

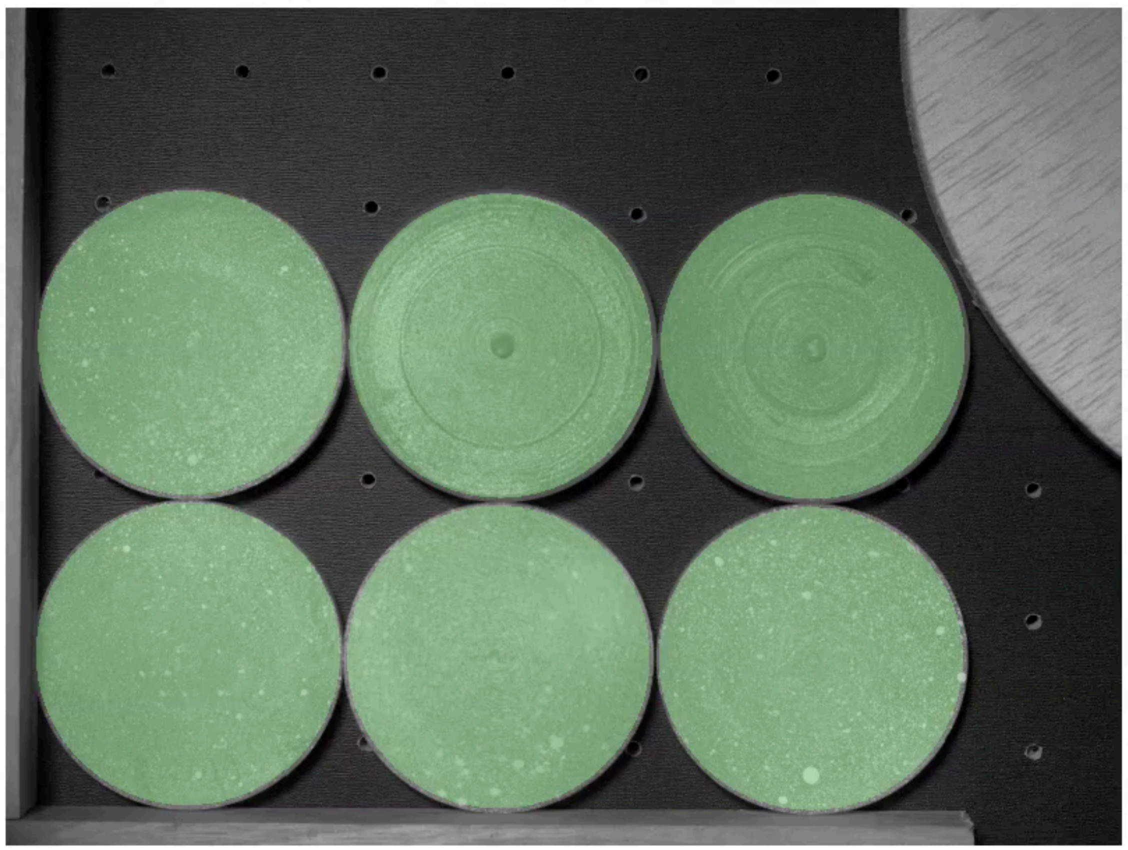
(1/500 real time)

(1/500 real time)



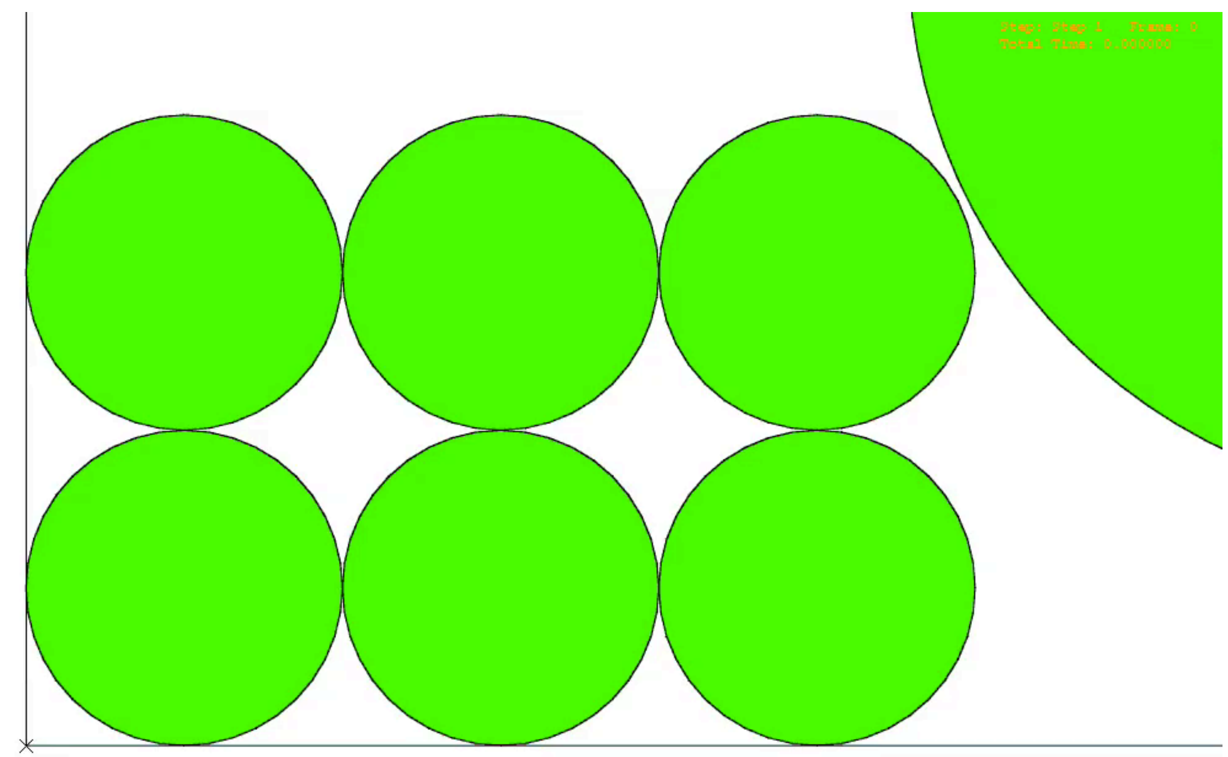
Dynamic Validation of Method

Digital Image Correlation (Vic2d): ϵ_{xy}



(1/500 real time)

FEM Simulation (Abaqus/Explicit): ϵ_{xy}



(1/500 real time)

$\epsilon_{ij} \leq -0.04$ $\epsilon_{ij} = 0$ $\epsilon_{ij} \geq 0.04$

Dynamic Validation of Method

Inverse problem

FEM Simulation

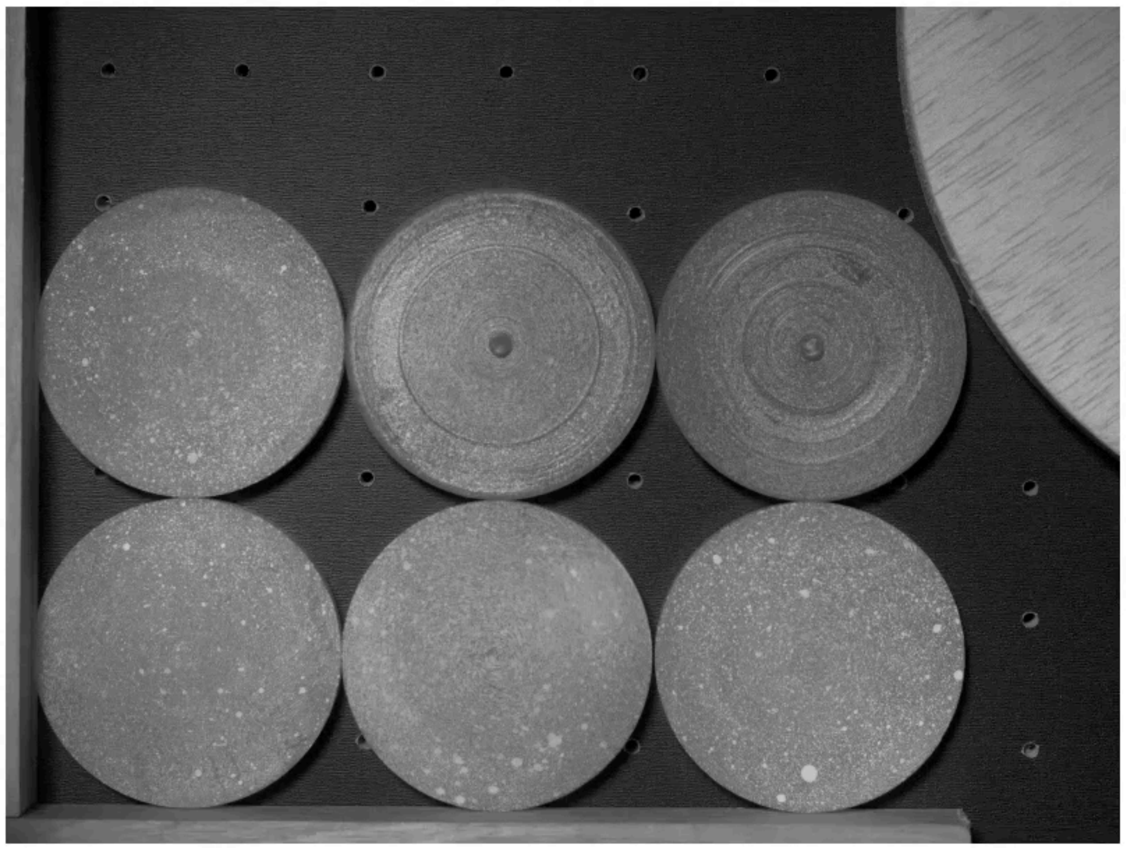
(1/500 real time)

25 N
20 N
15 N
10 N
5 N

(1/500 real time)

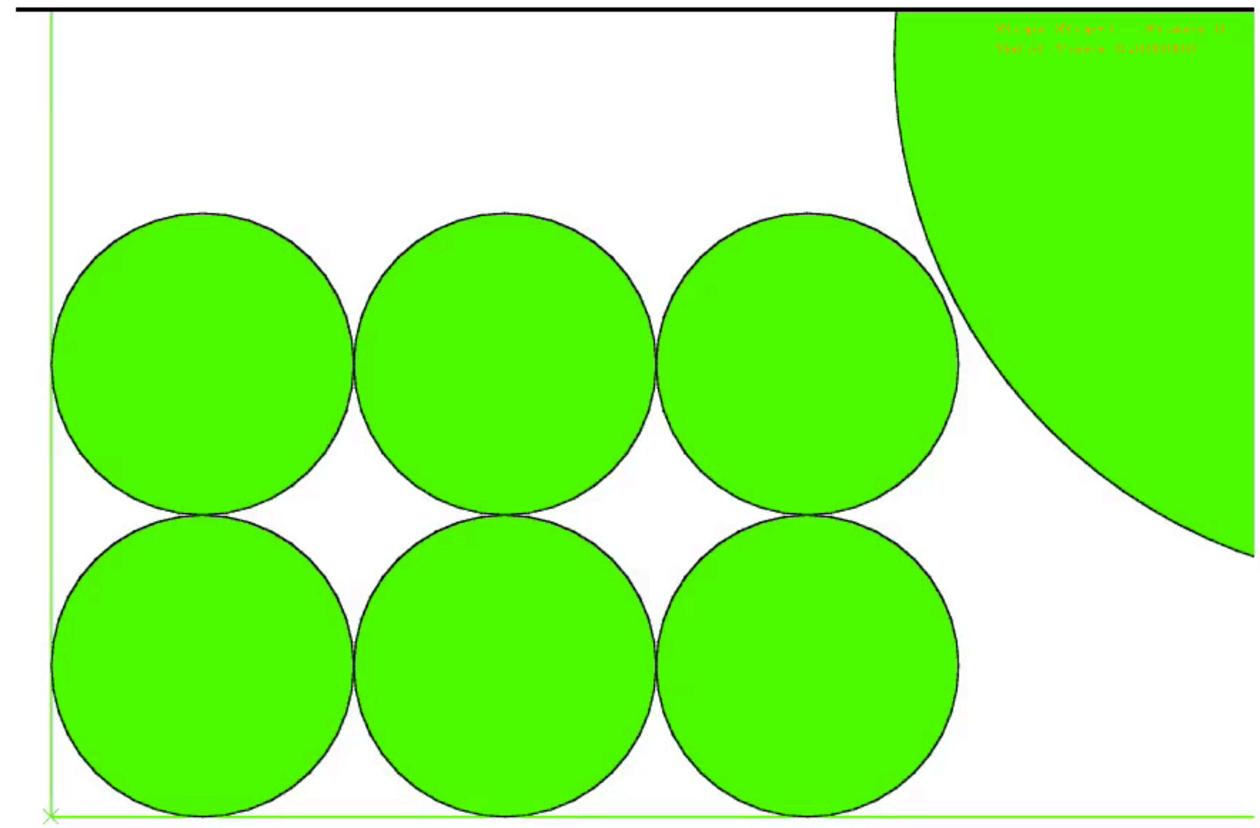
Dynamic Validation of Method

Inverse problem



(1/500 real time)

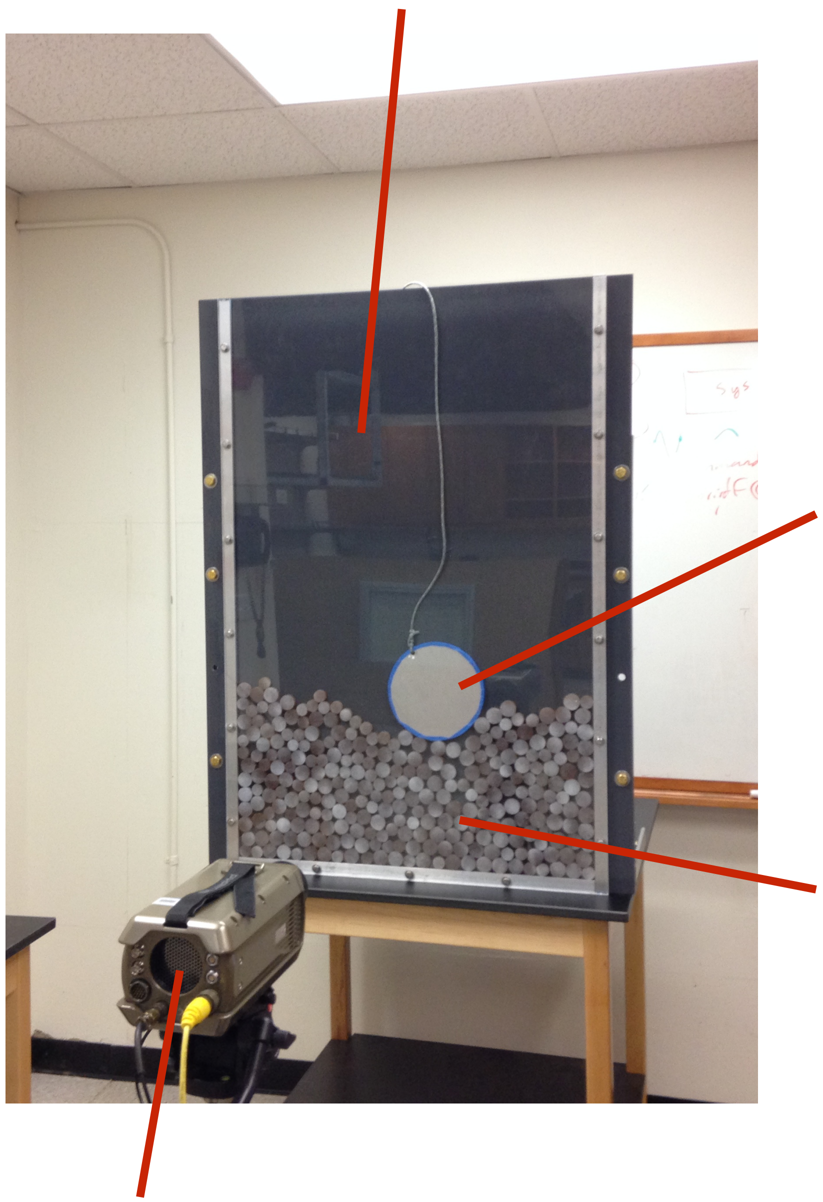
FEM Simulation



(1/500 real time)

25 N
20 N
15 N
10 N
5 N

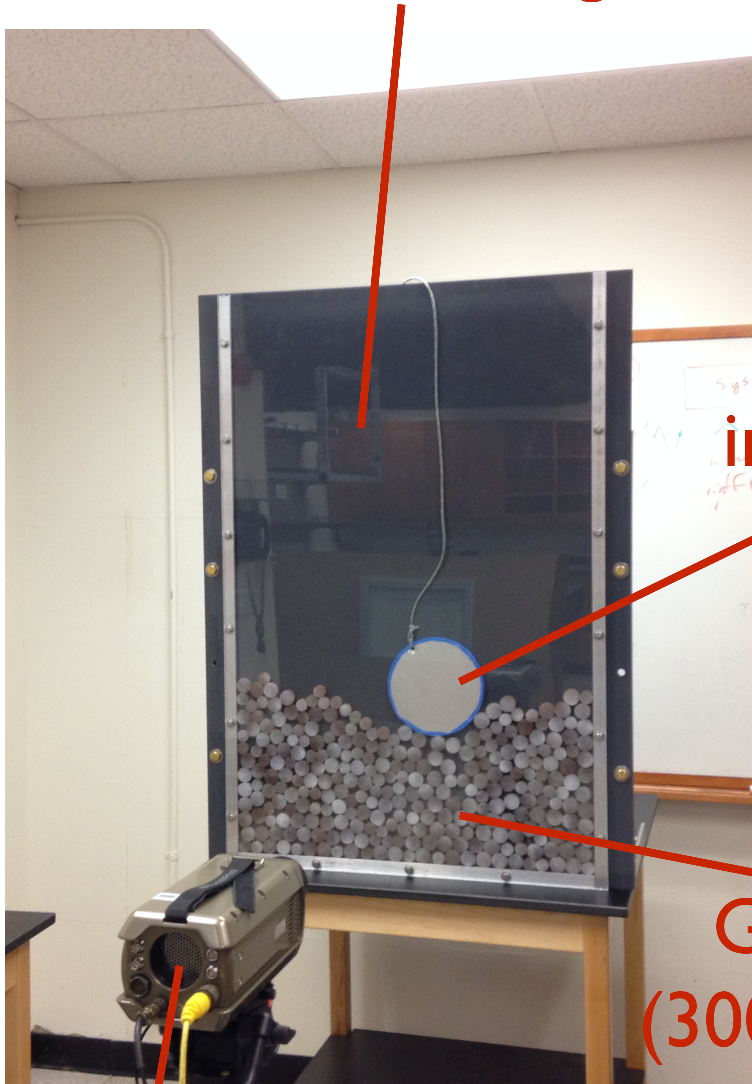
Application: Penetration



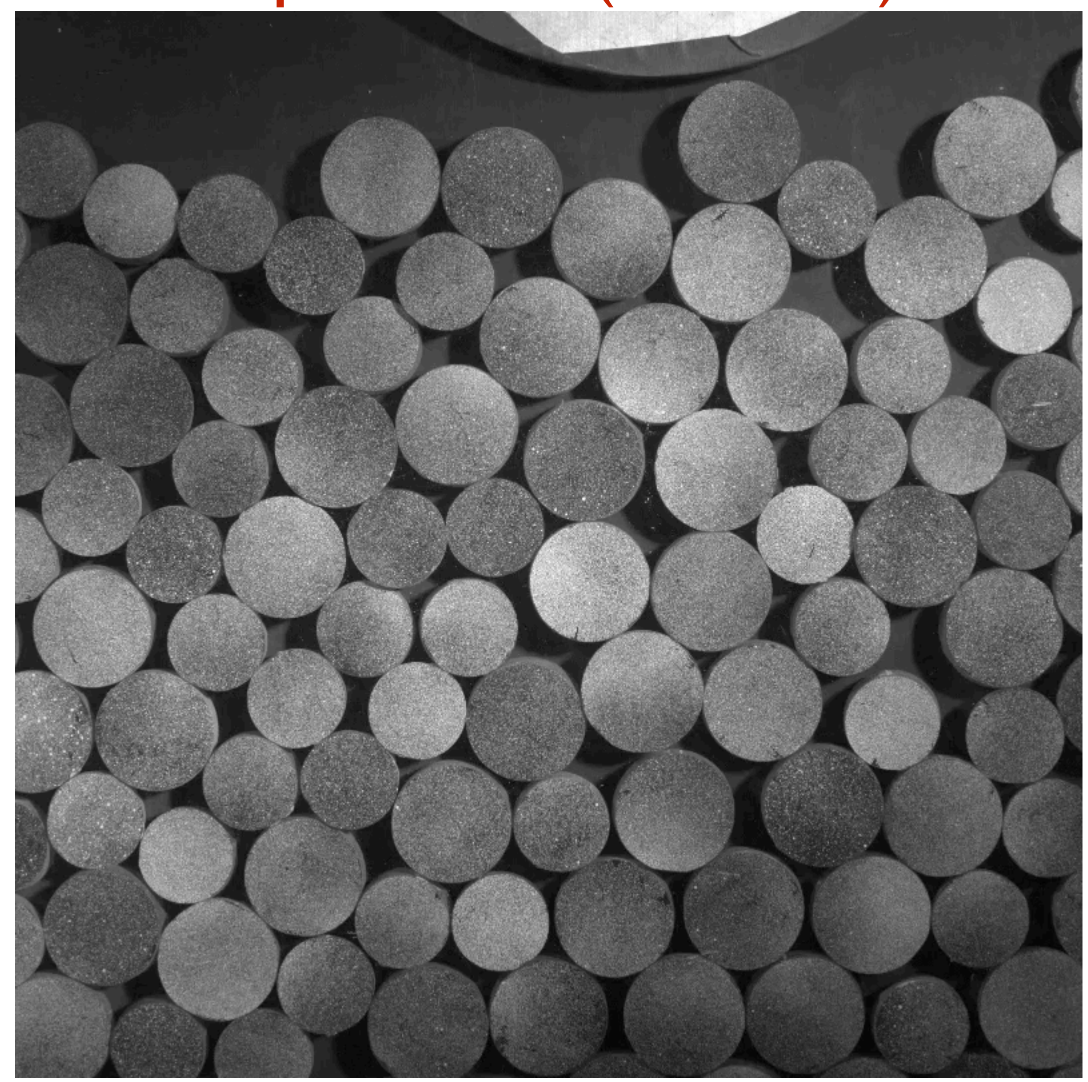
Experiment (v_0=3m/s)

(1/450 real time)

Application: Penetration



Experiment (v_0=3m/s)



(1/450 real time)

Application: Penetration

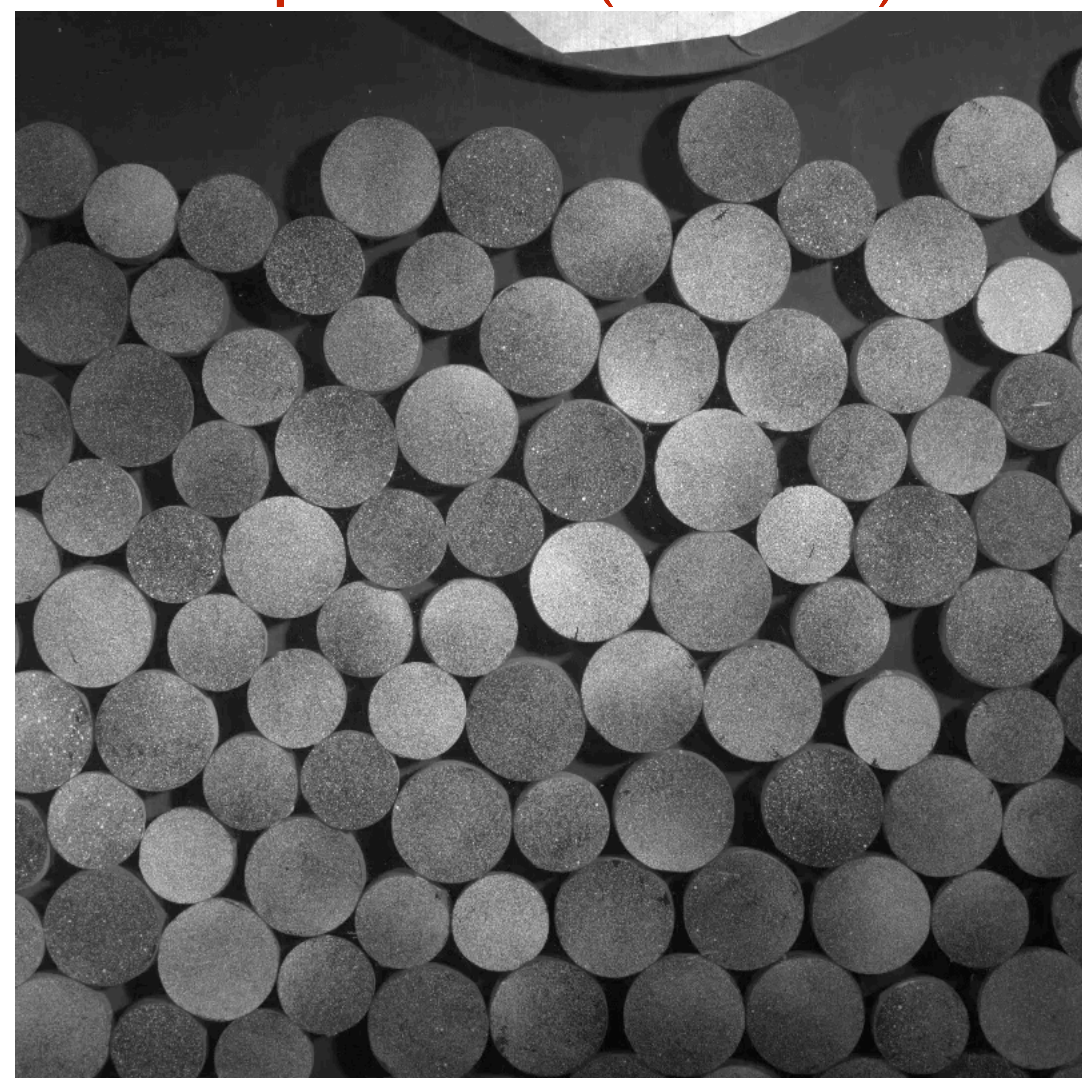
PVC & Plexiglass

Steel intruder

Grains (300 - 600)

Phantom v310 high-speed camera

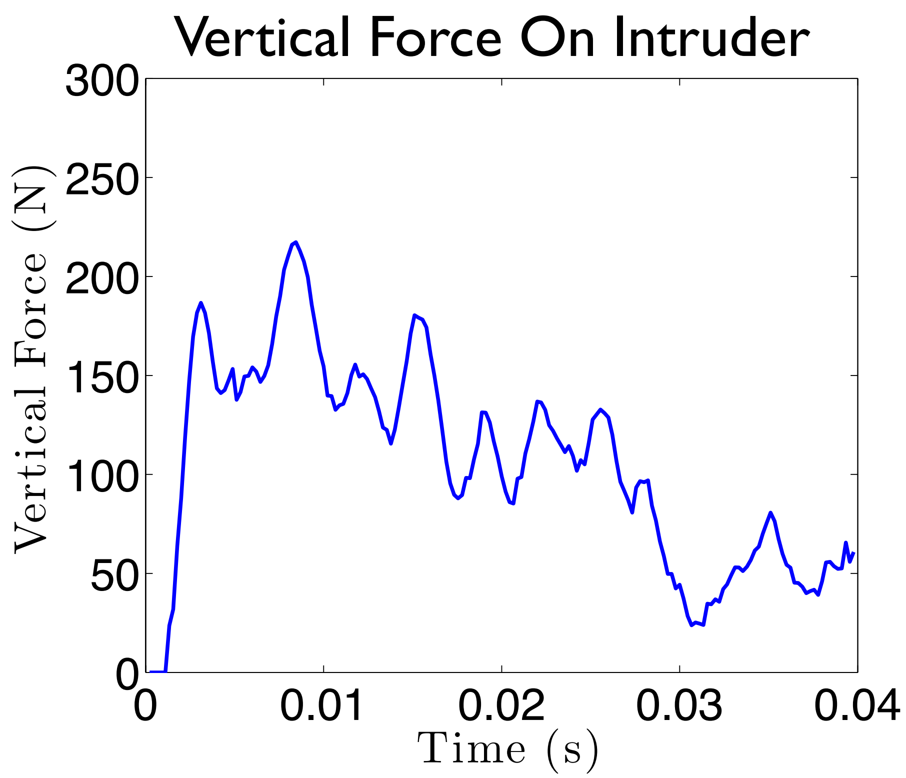
Experiment (v_0=3m/s)



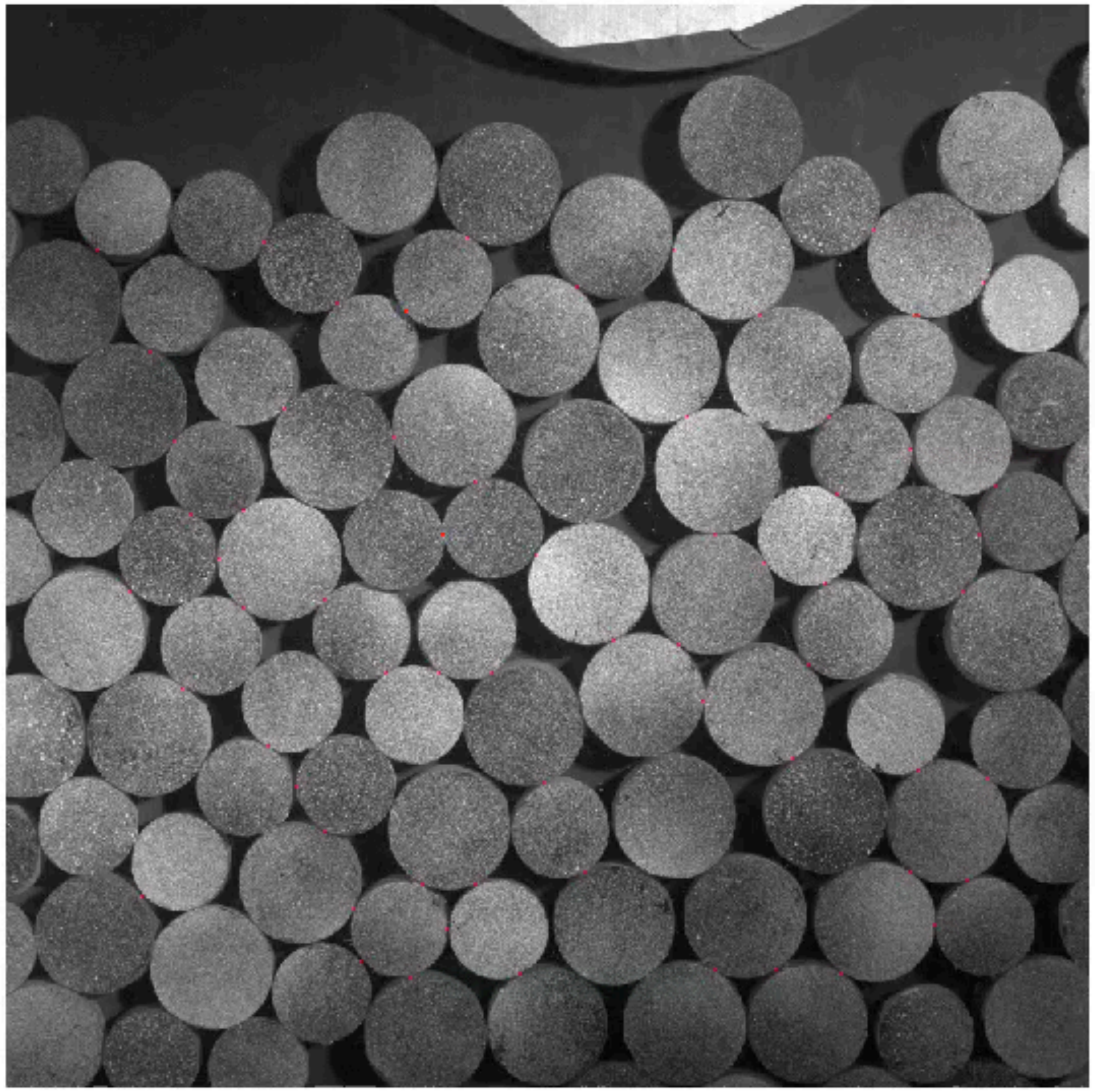
(1/450 real time)

Application: Penetration

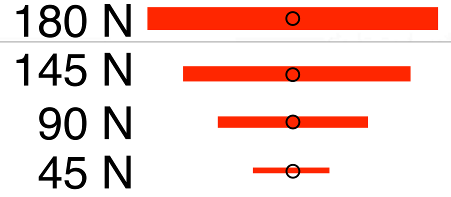
Application: Penetration



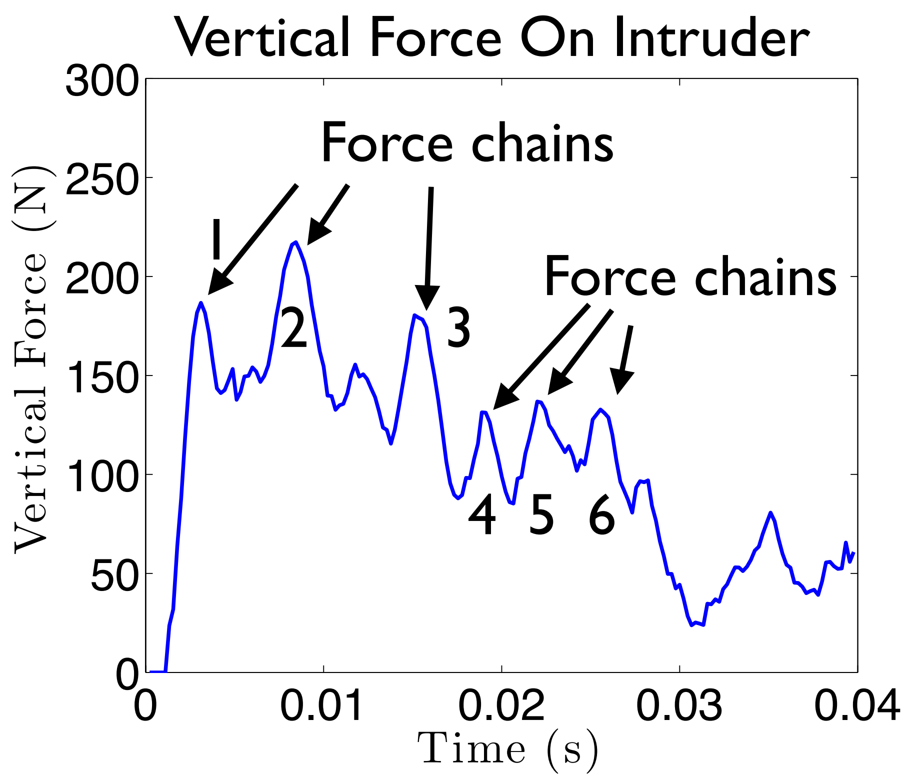
Inverse problem



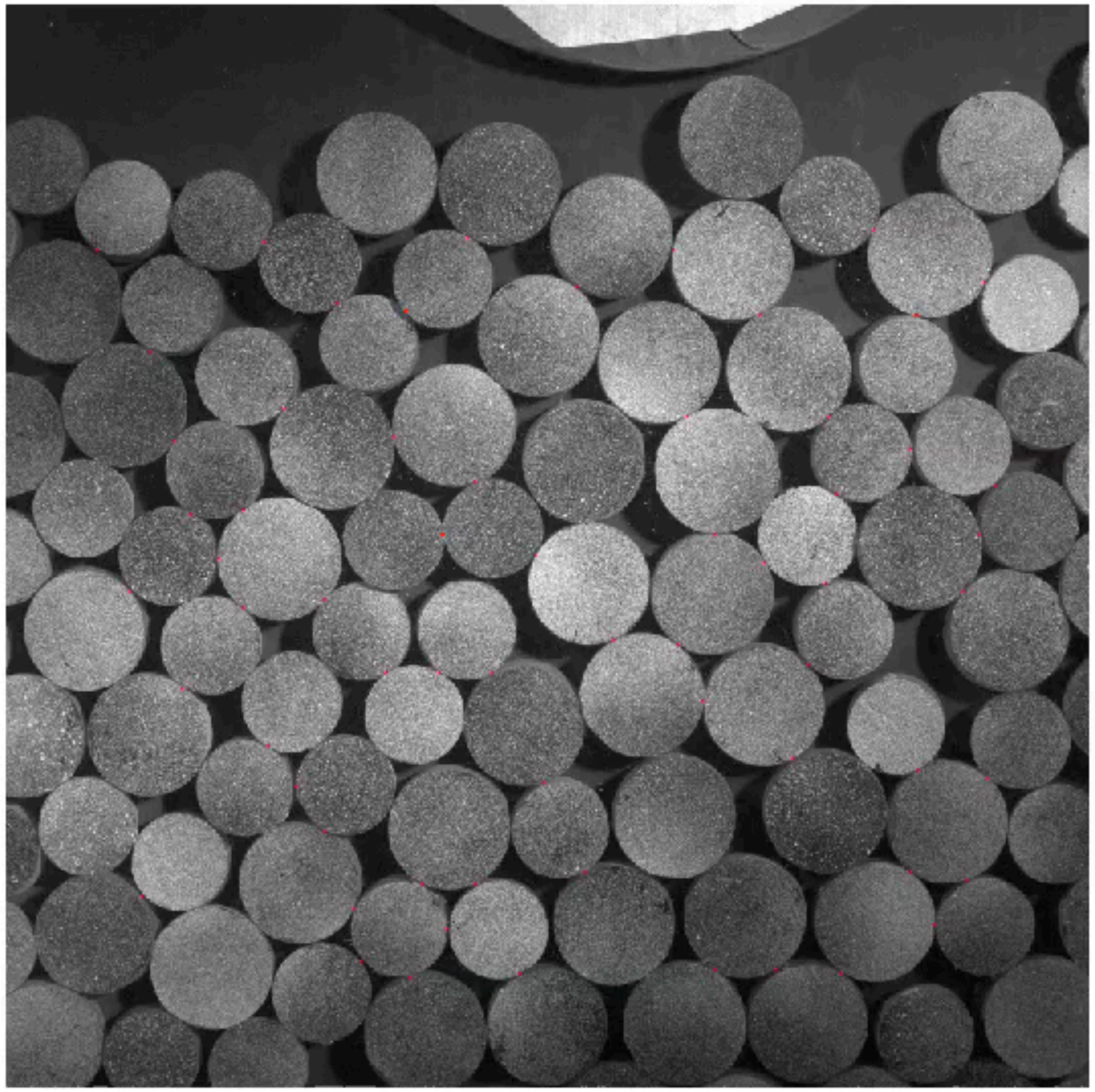
(1/450 real time)



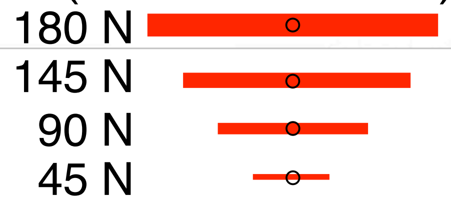
Application: Penetration



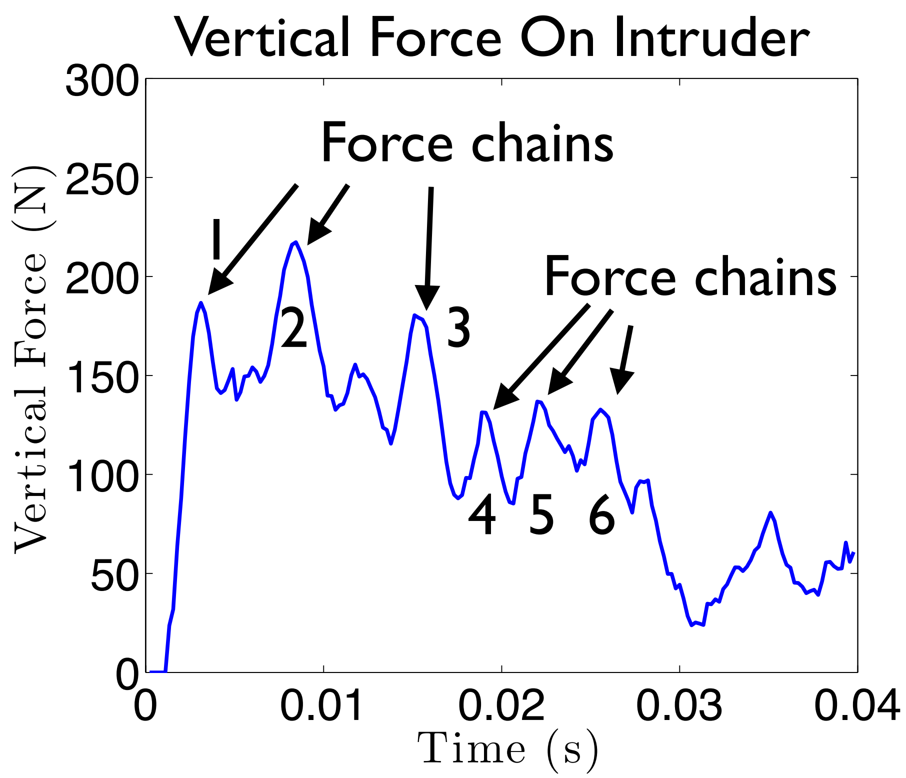
Inverse problem

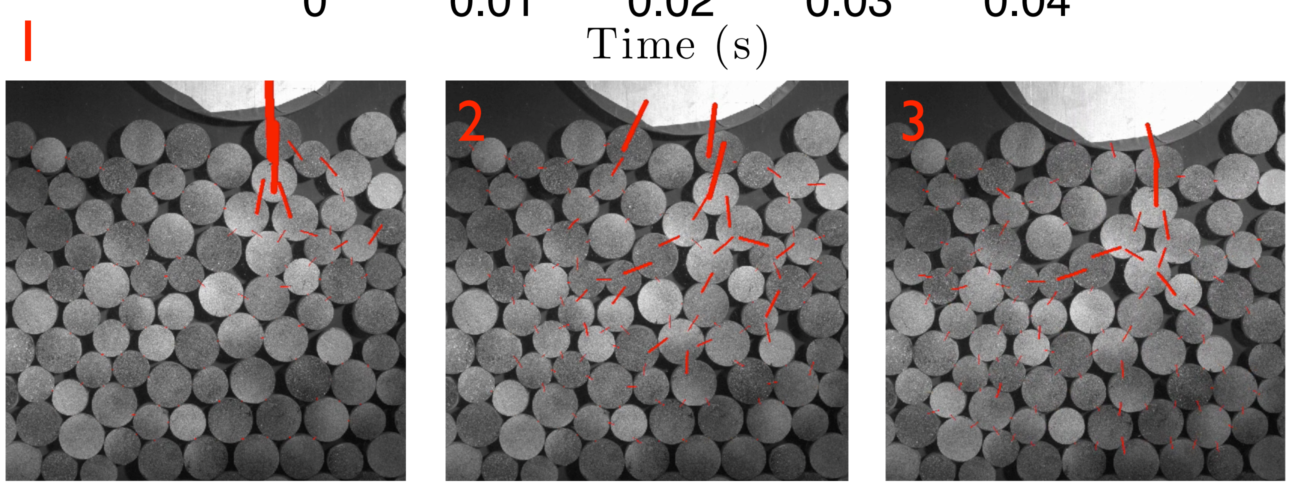


(1/450 real time)

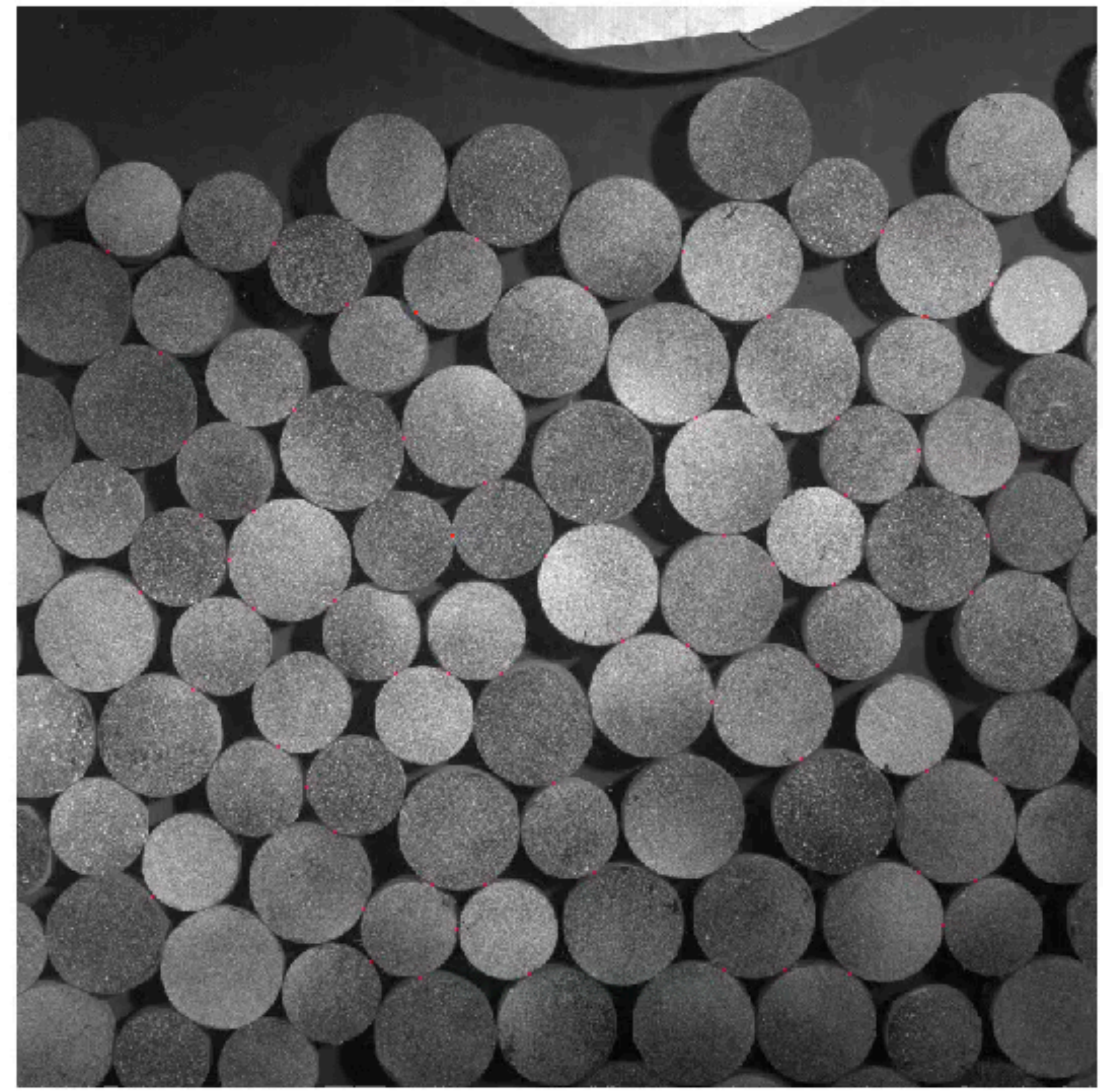


Application: Penetration



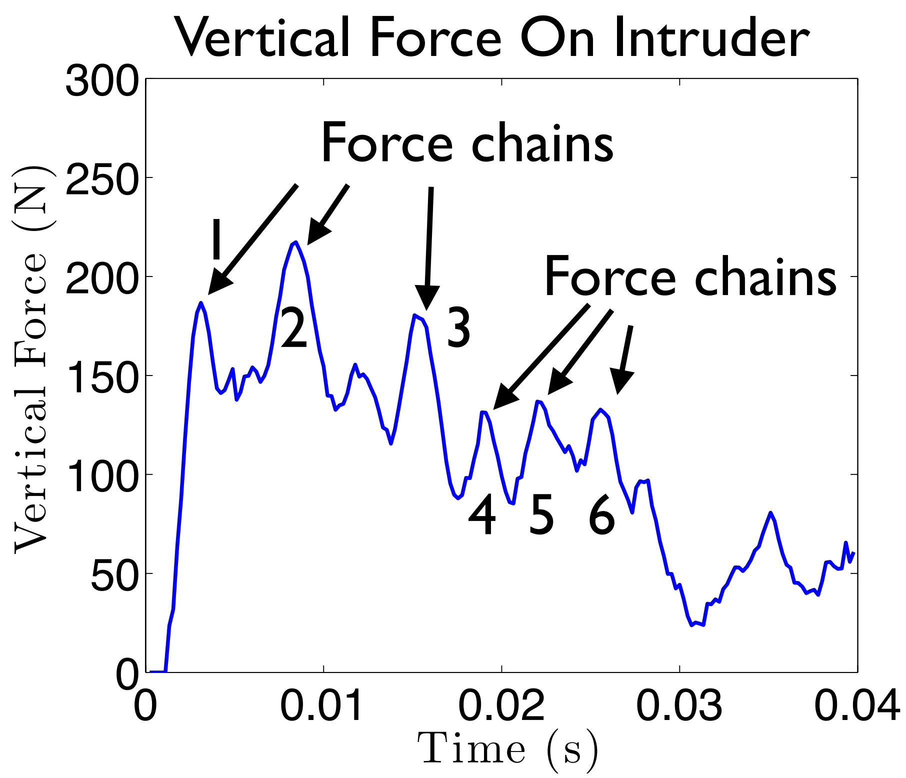


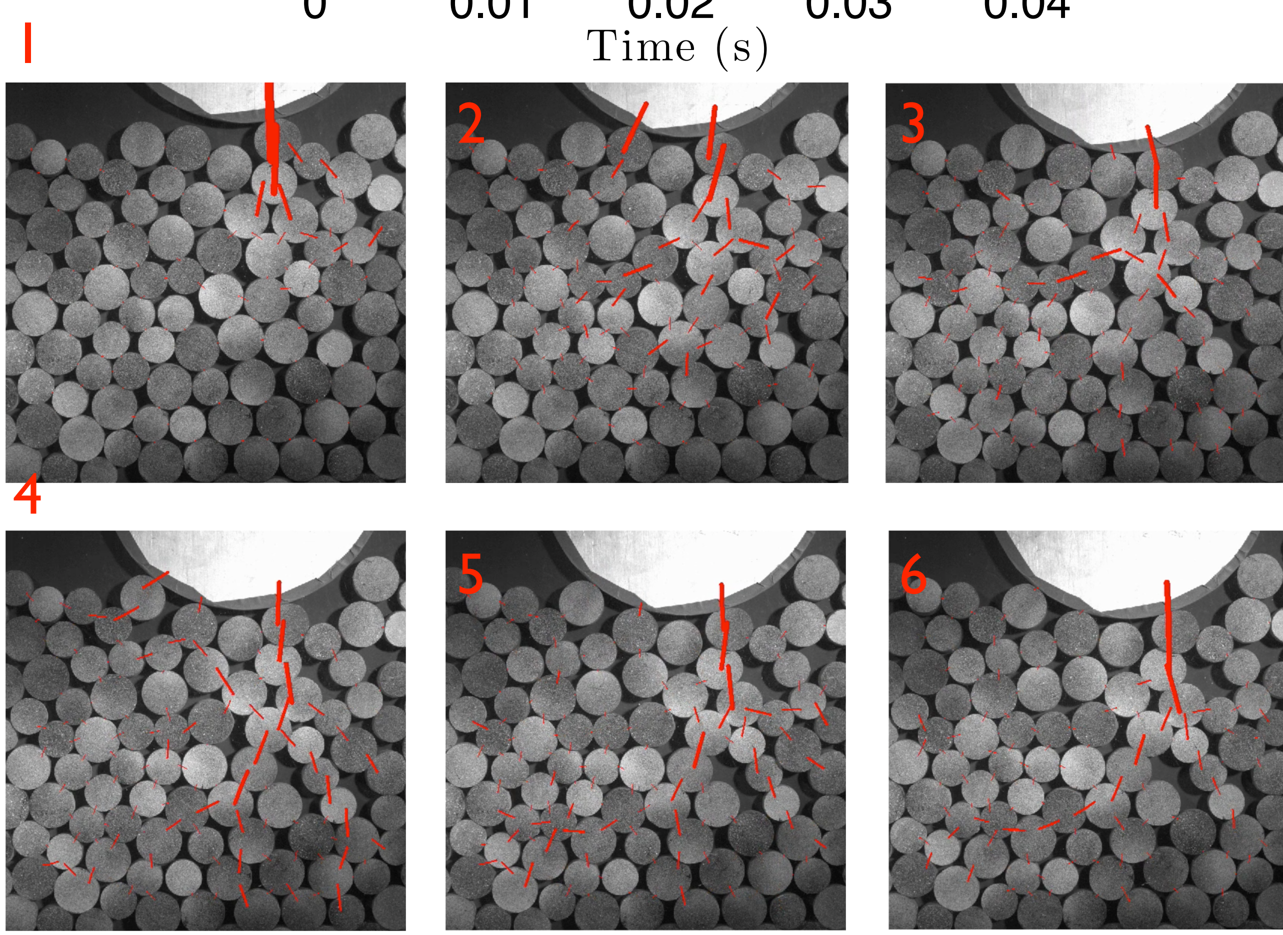
Inverse problem



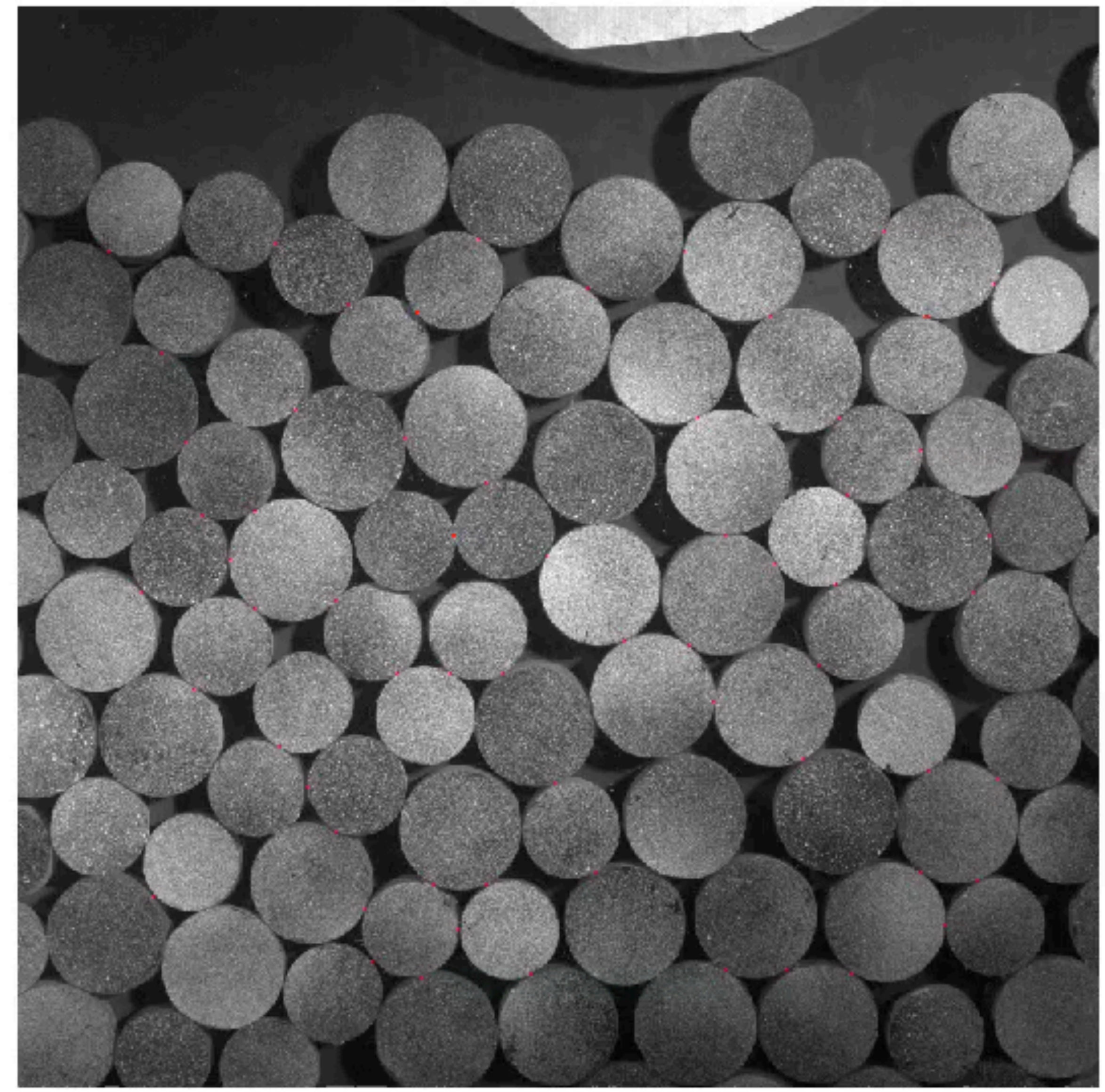
(1/450 real time)

Application: Penetration





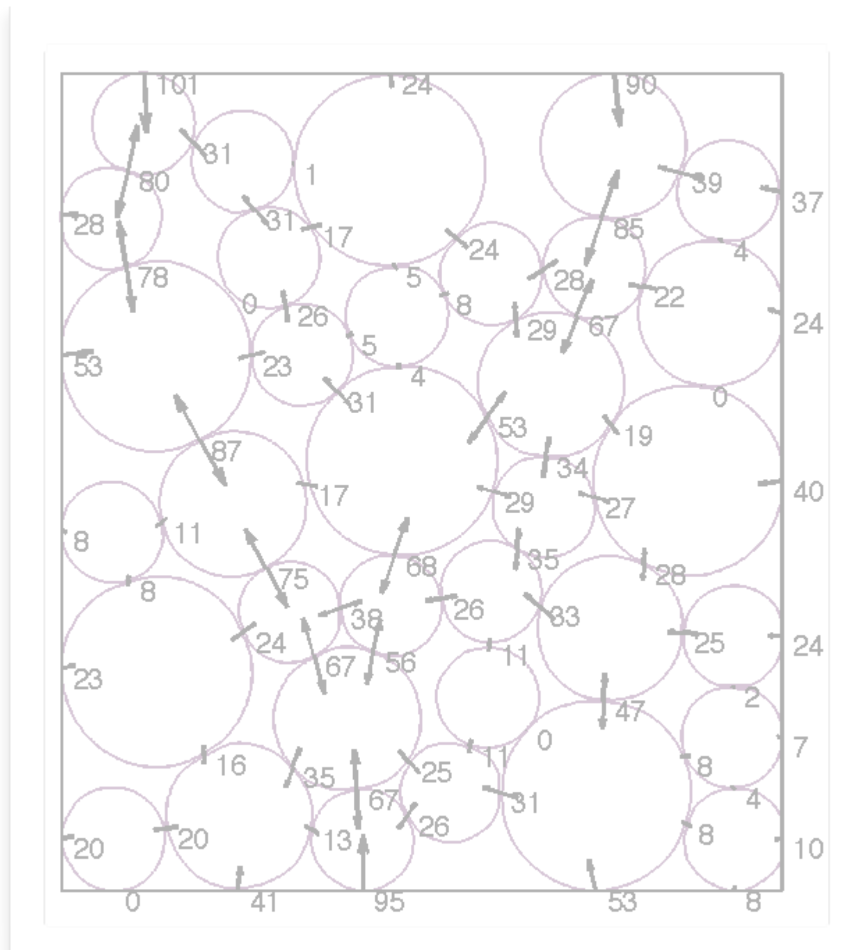
Inverse problem



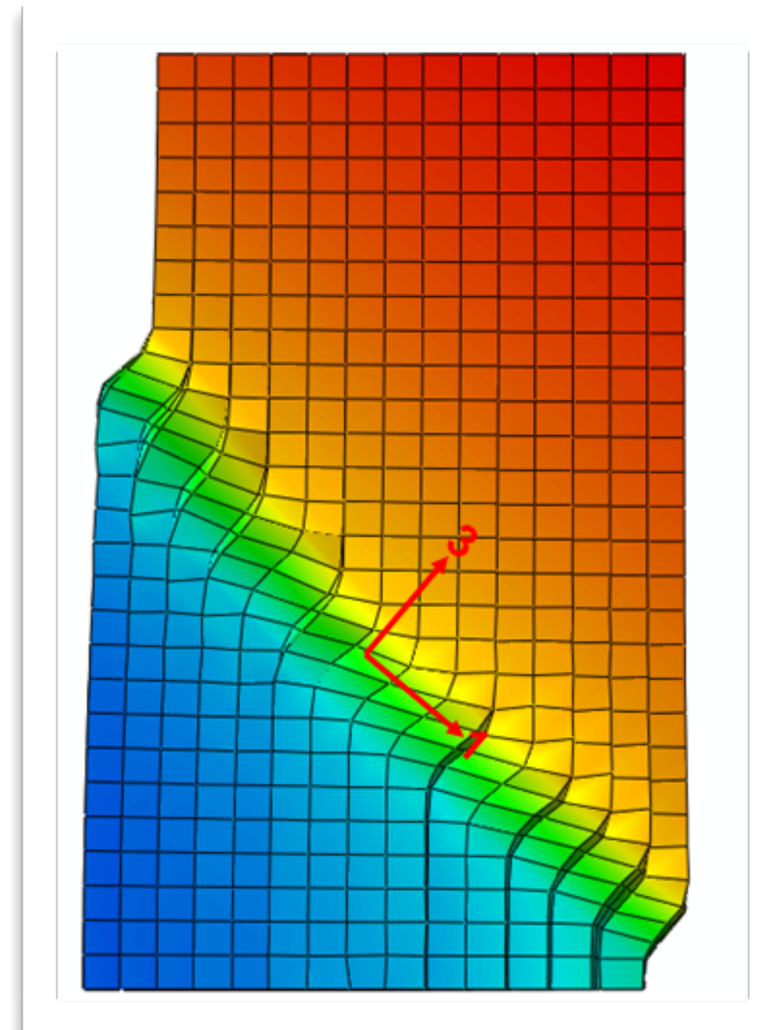
(1/450 real time)

Today's menu: Avatar

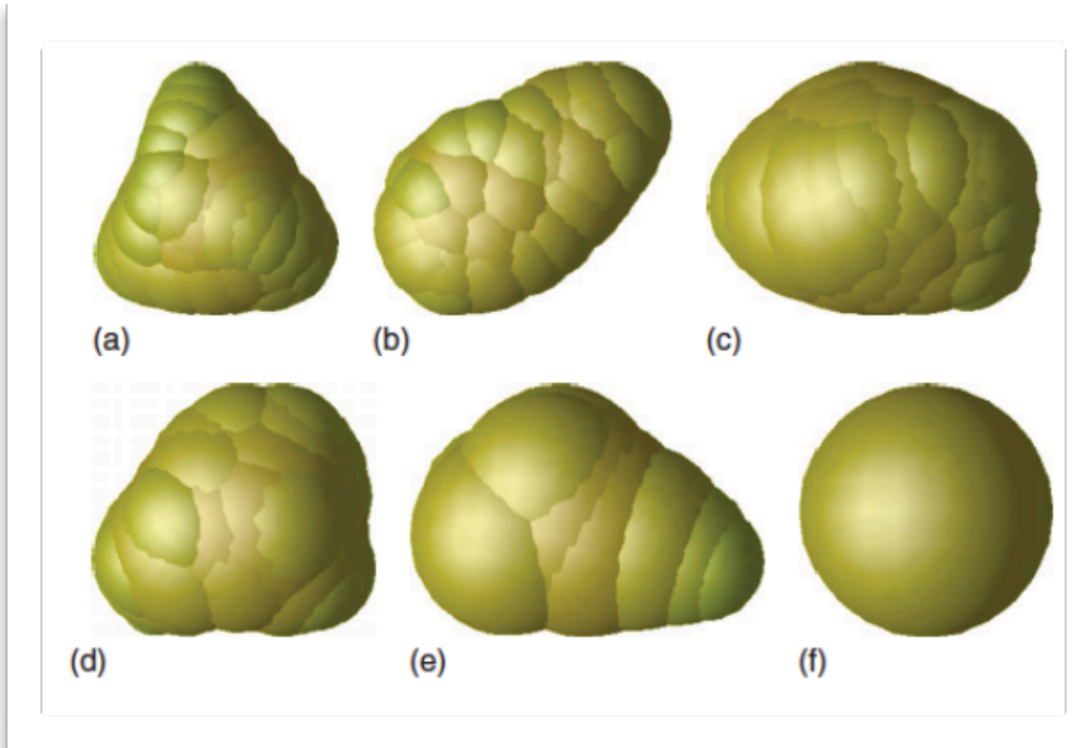
measuring kinematics



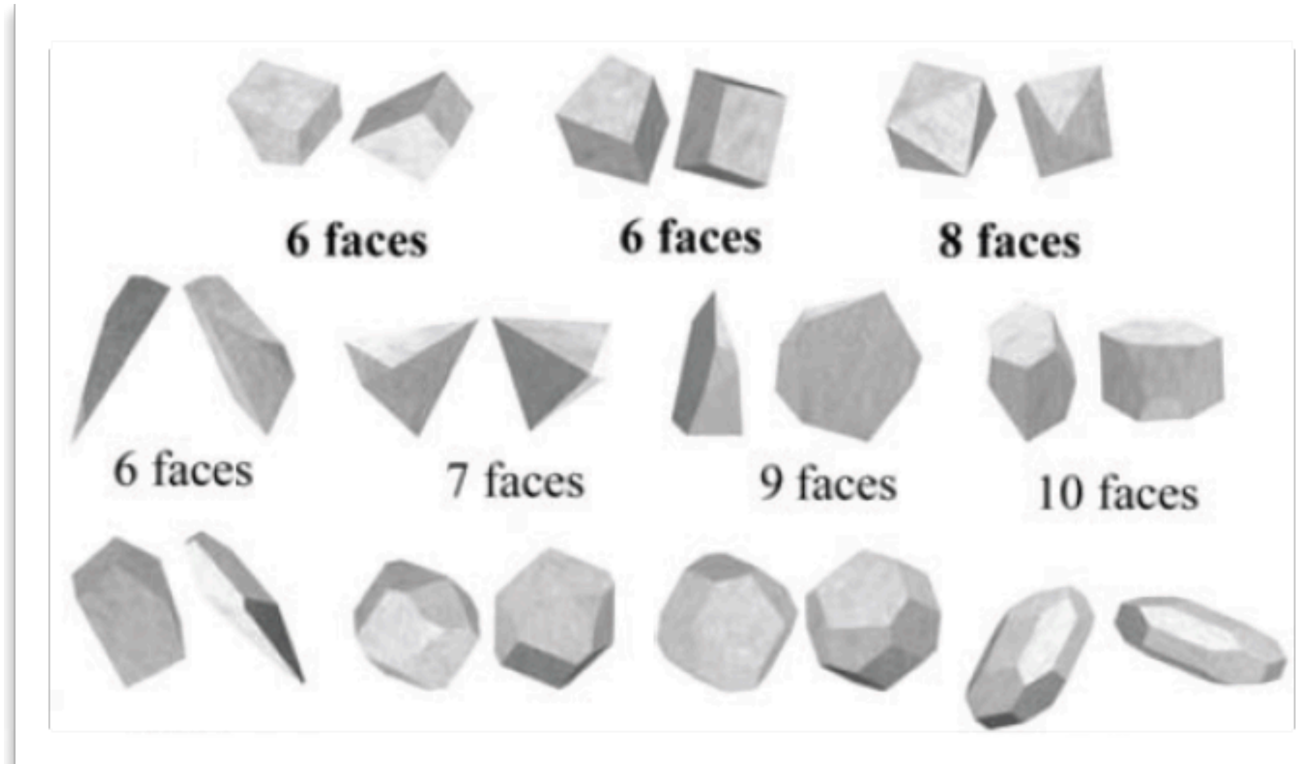
measuring forces



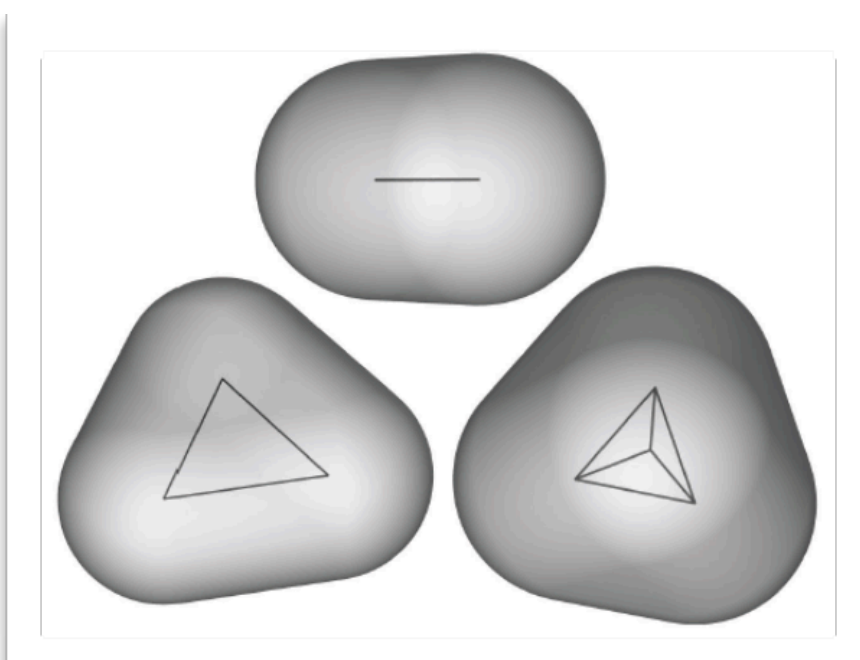
predicting behavior

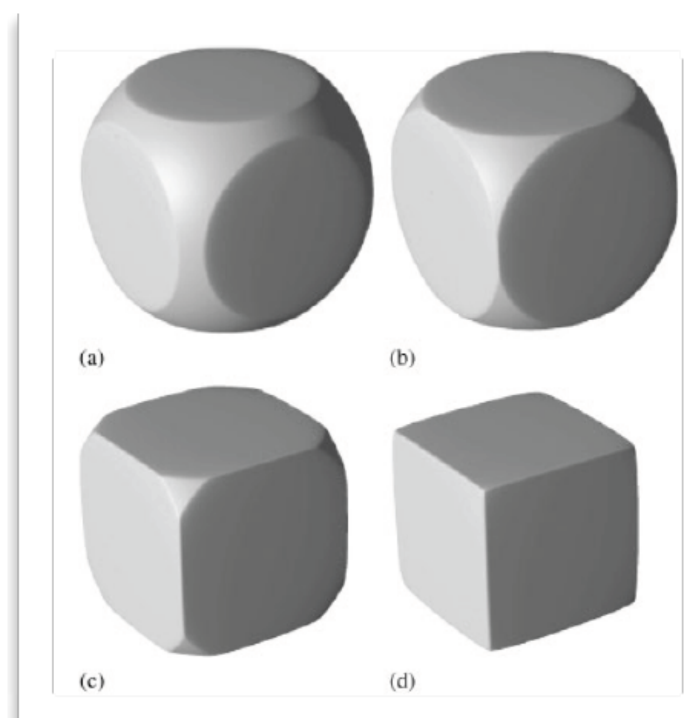


Clustering (using spheres)



Polyhedra

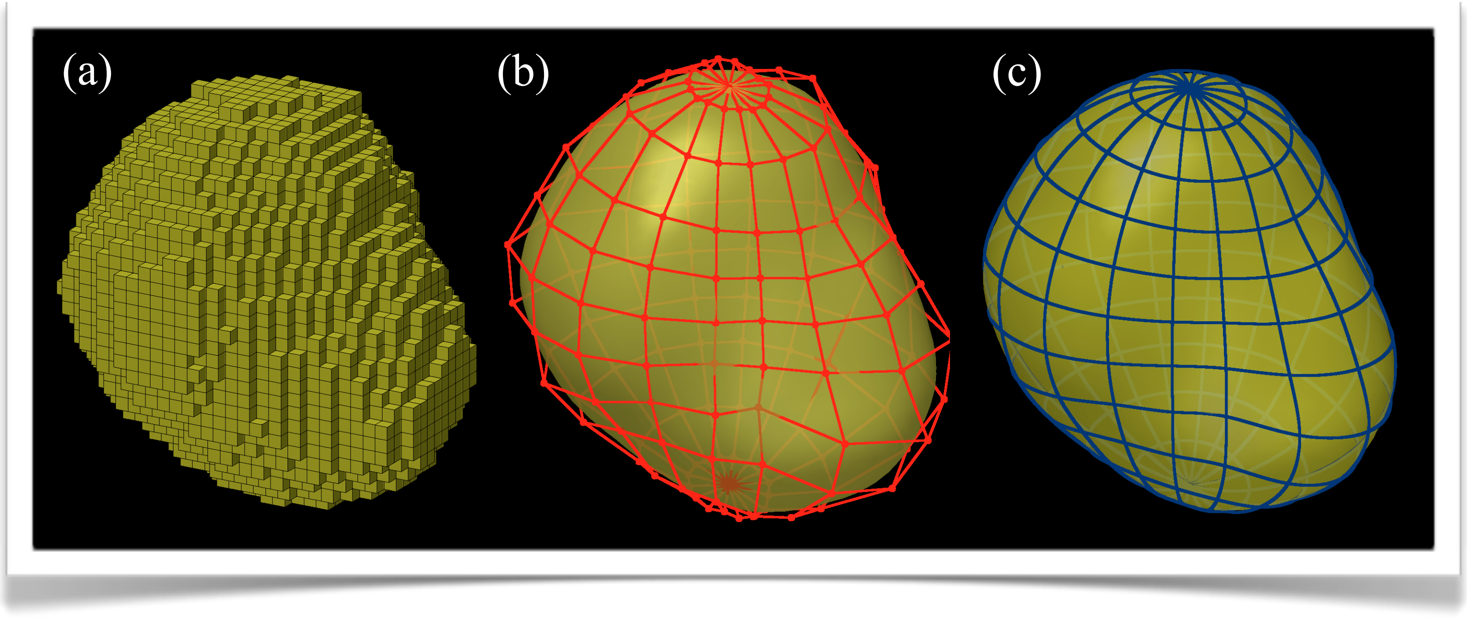

Spheropolyhedra



Potential Particles

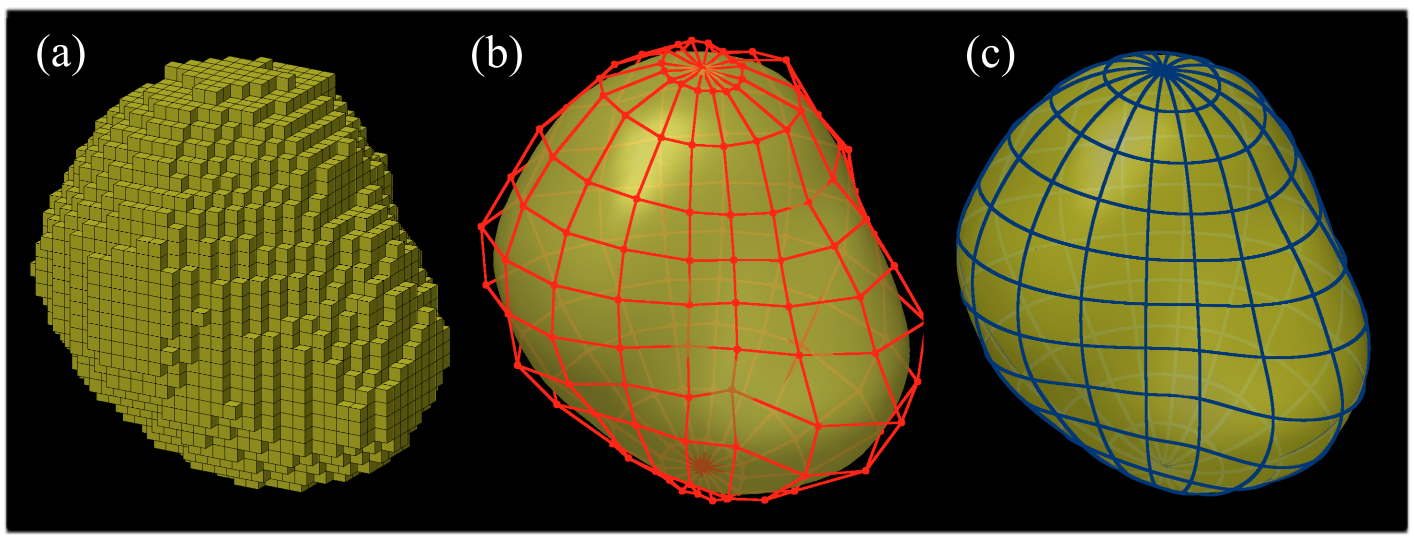
Granular Element Method

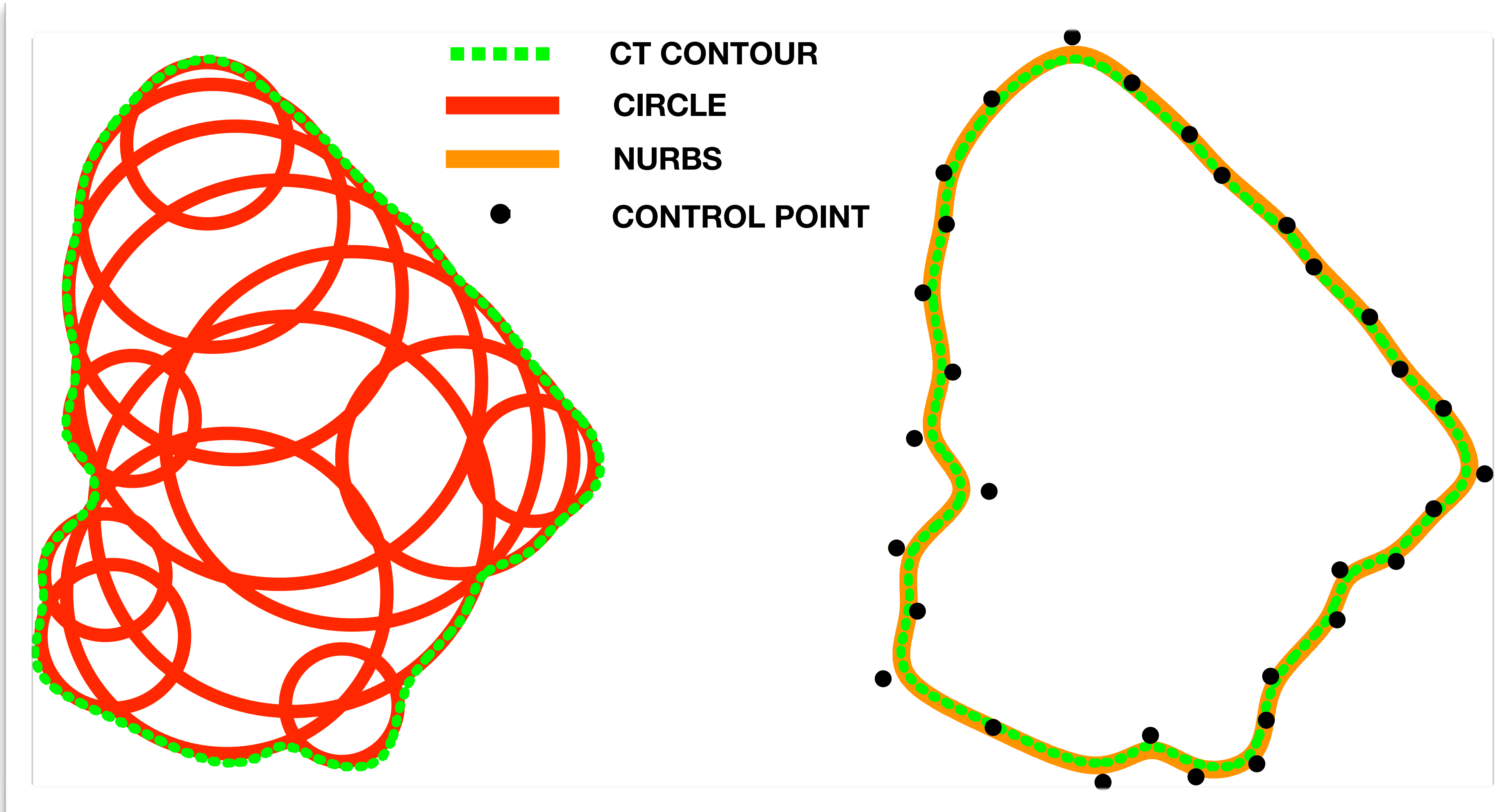
- A new discrete element method that has the ability to represent complicated grain geometries
- Uses parametric curves and surfaces
- Brings free-form computer-aided graphics design technology to full force for the realistic simulation of granular materials

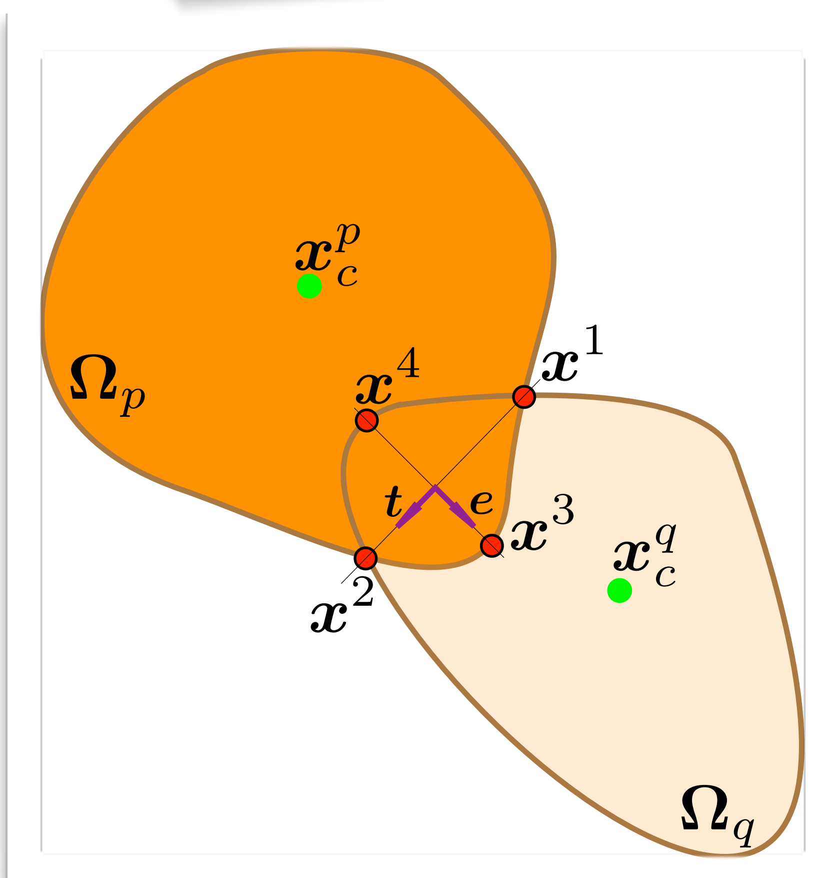


Features of GEM

- Captures particle morphology: sphericity & roundness
- Can seamlessly go from binary images (XRCT) to model
- Can perform calculations with real shapes
- Accurately predicts macroscopic response
- Retains the simplicity of conventional DEM

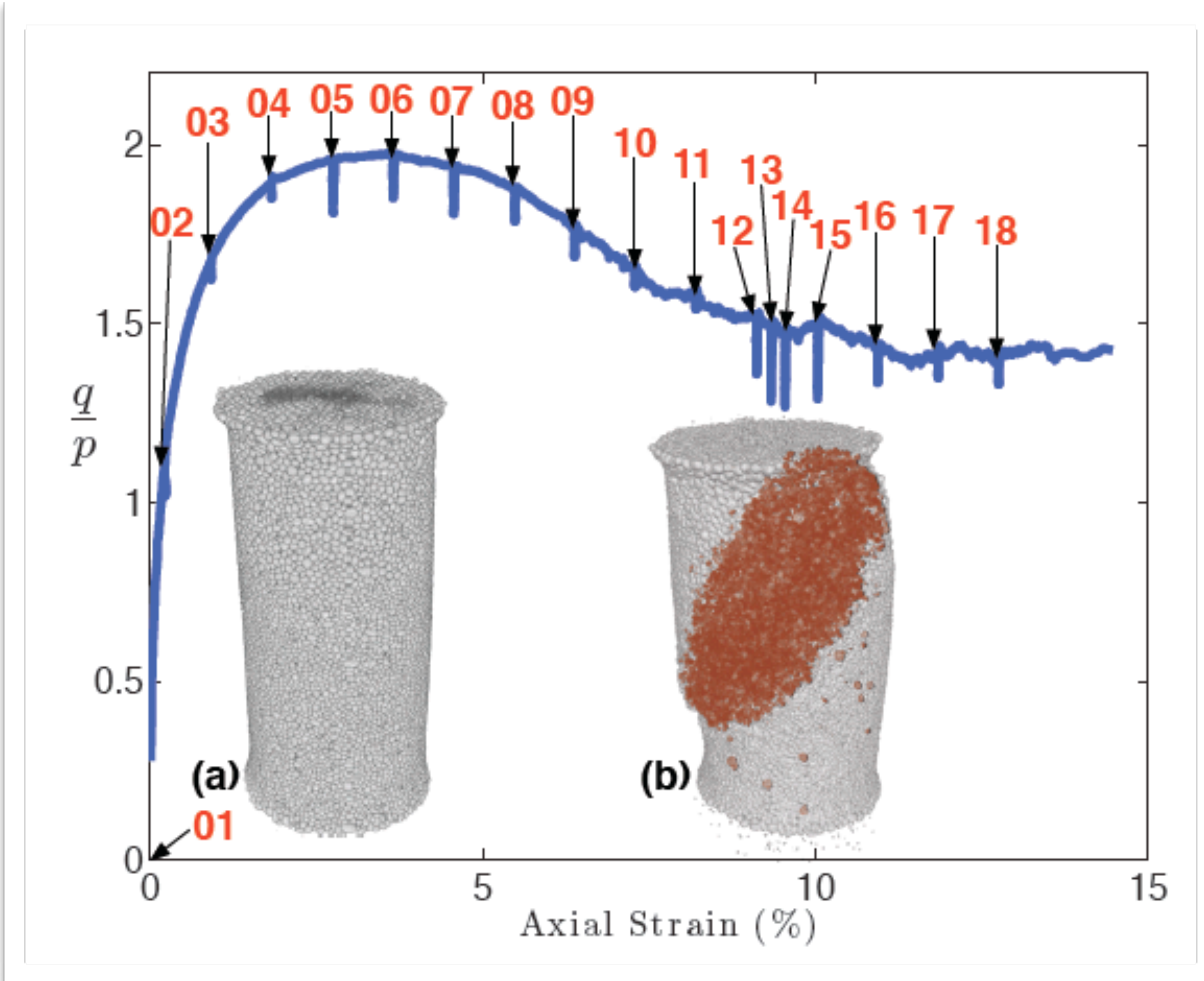




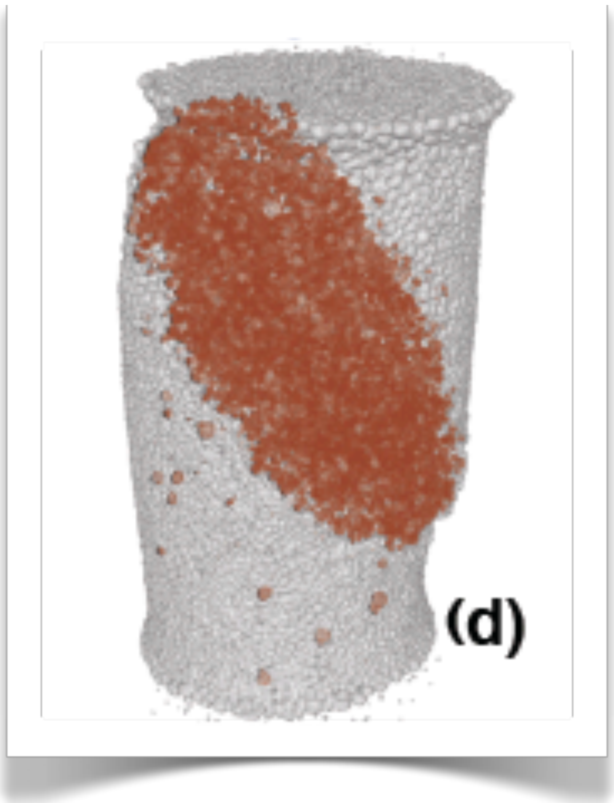


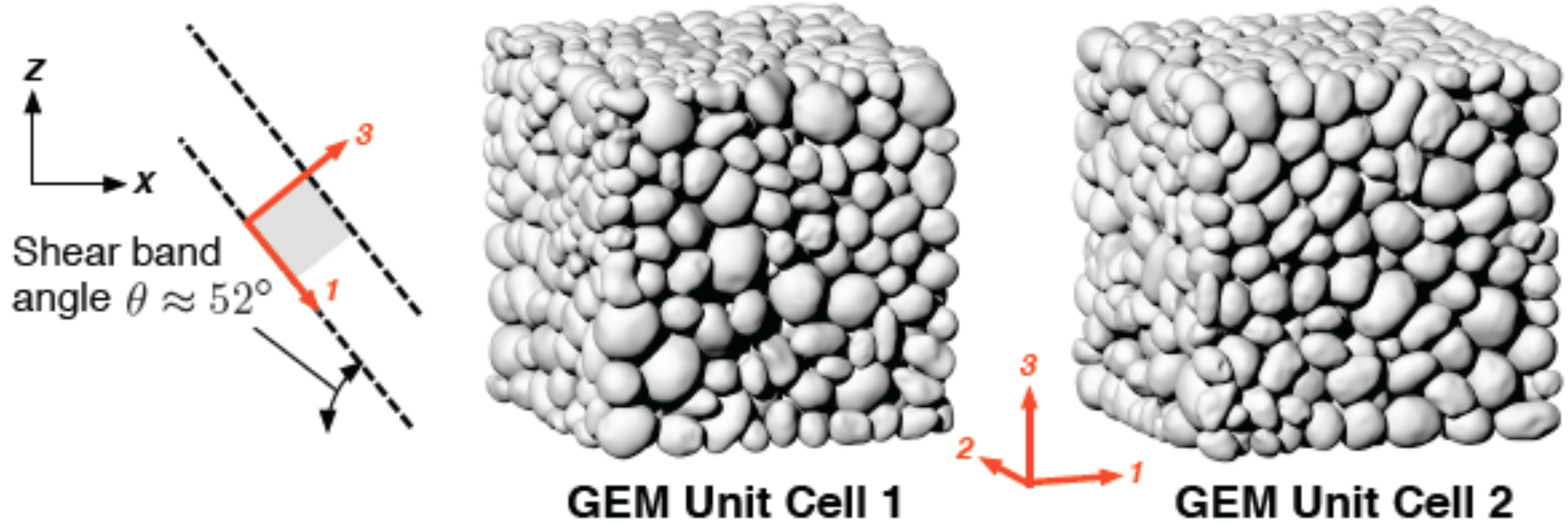
$$
\begin{aligned}
ma_i + Cv_i &= F_i \\
I\alpha + \bar{C}\omega &= M
\end{aligned}
$$

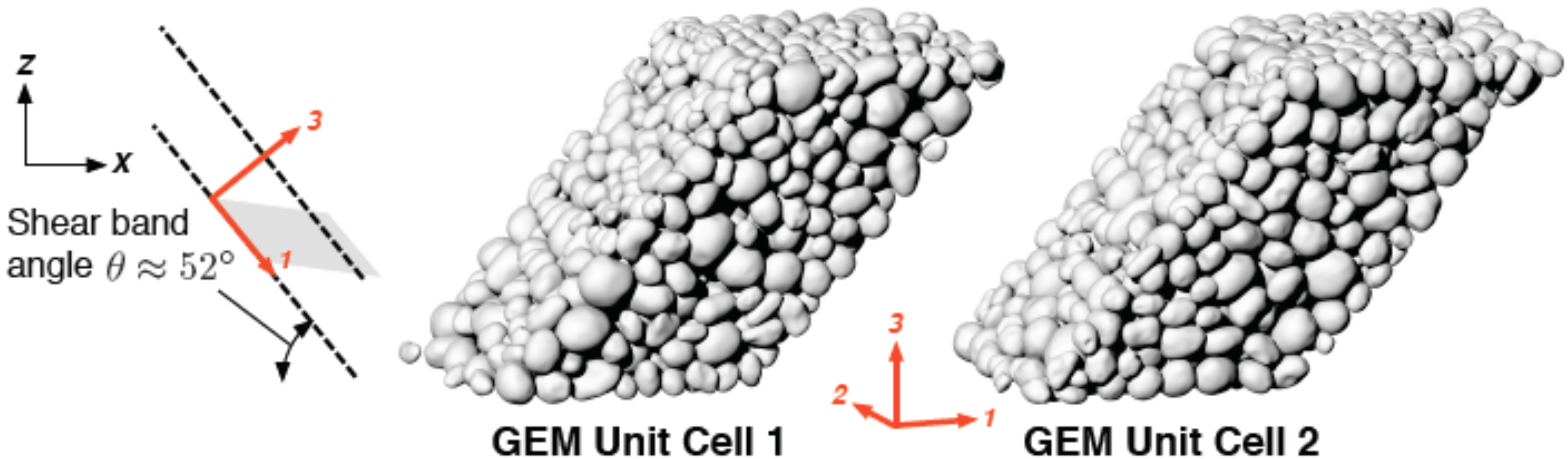
forces and moments depend on particle morphology



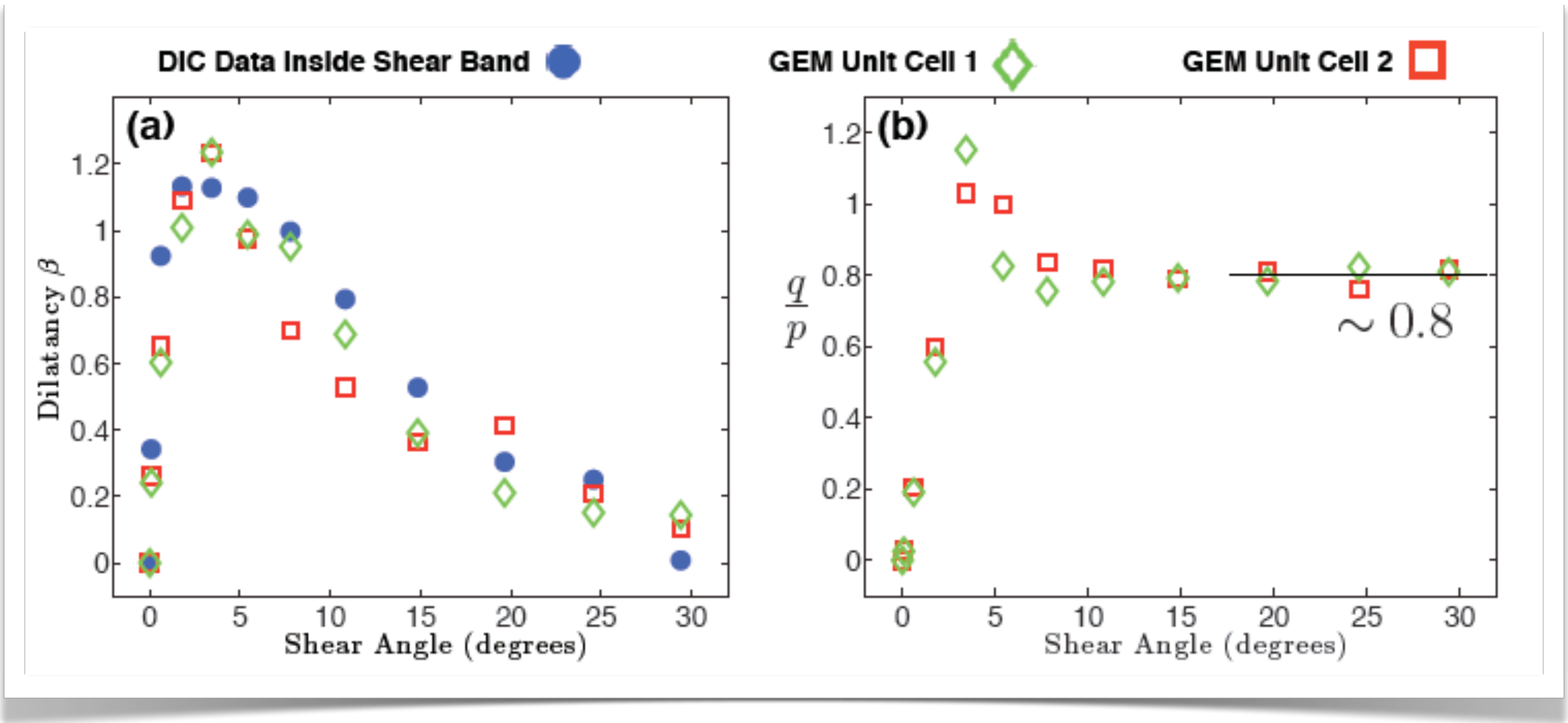
Experimental data: stress-strain (macro) & grain kinematics (micro)







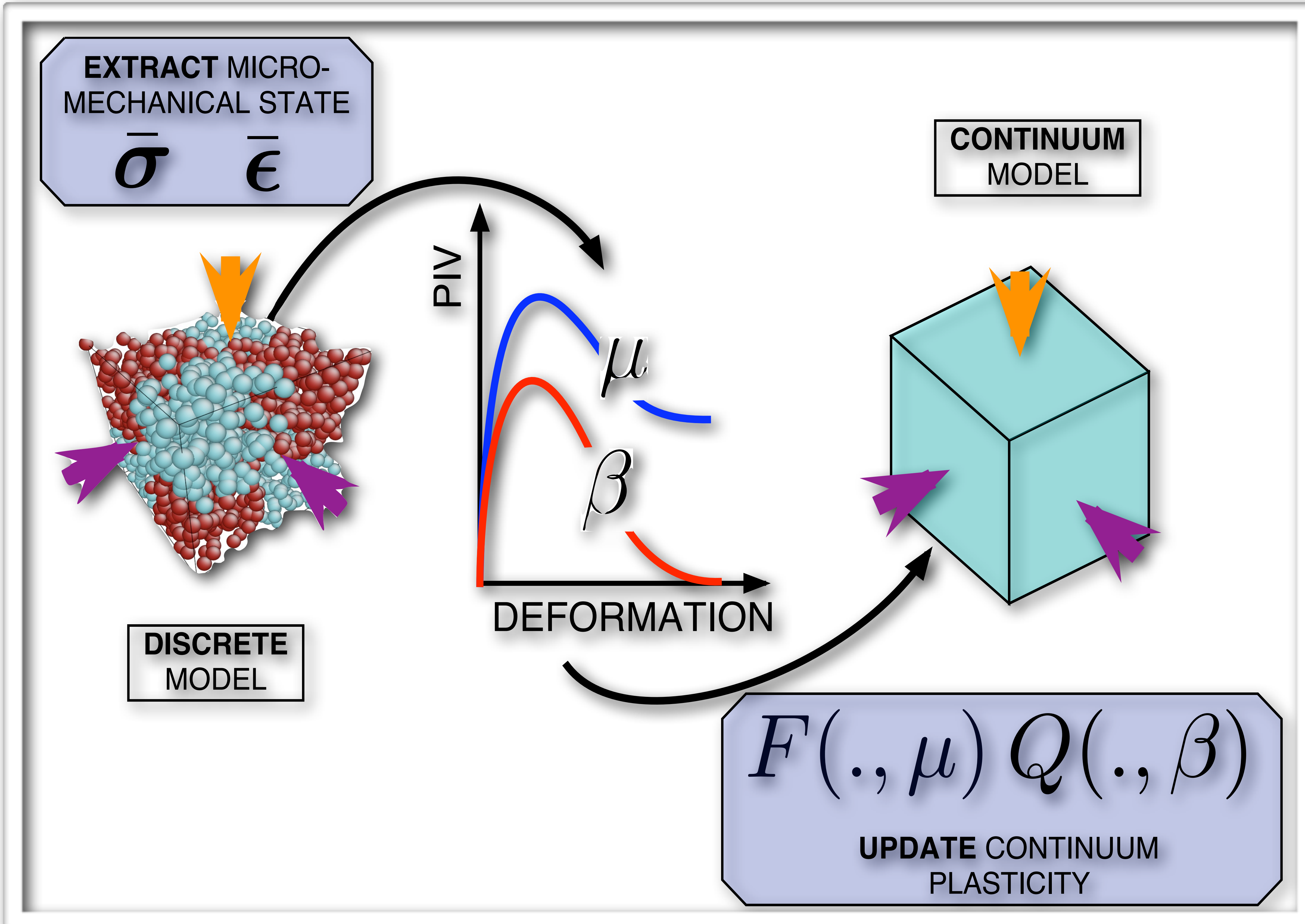
Simple shear numerical experiments



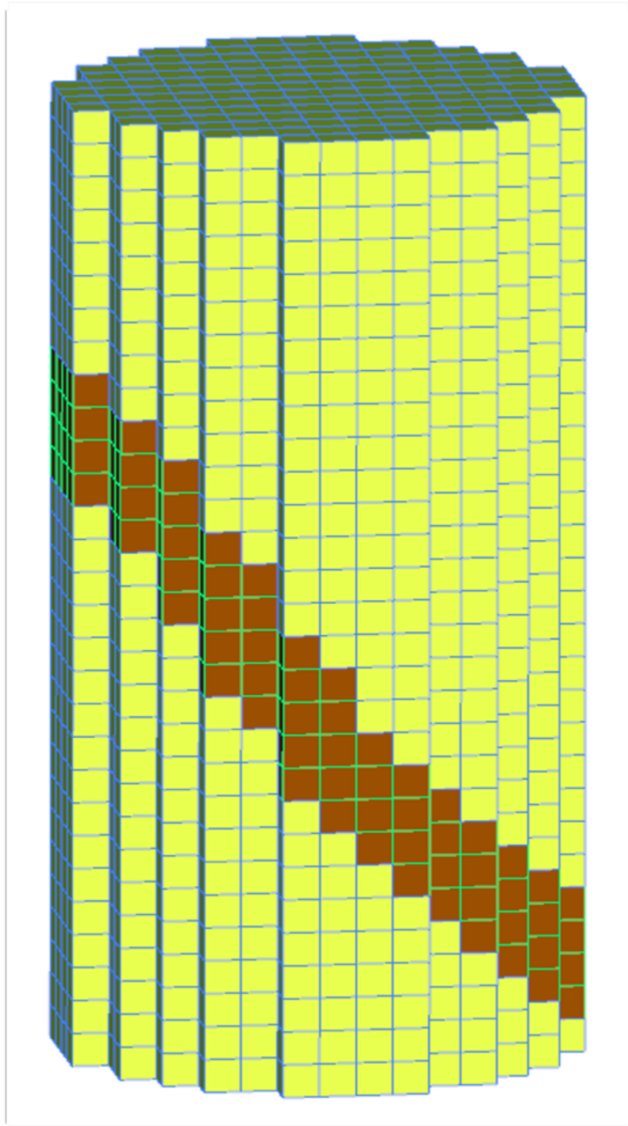
$$F(p, q, \mu) = q + \mu p = 0$$

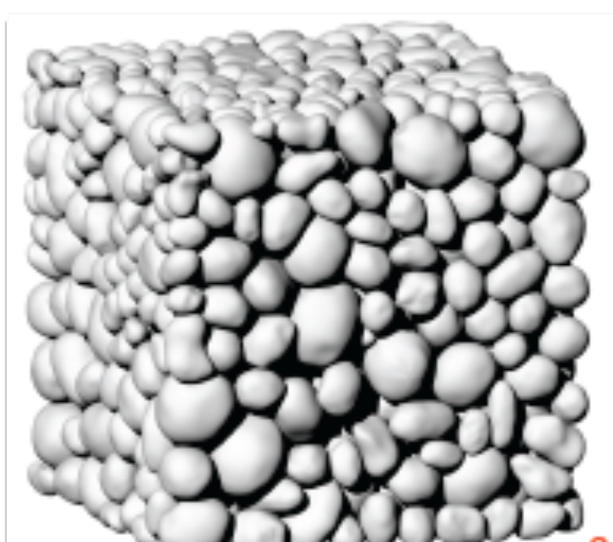
$$Q(p, q, \beta) = q + \beta p - \bar{c} = 0$$

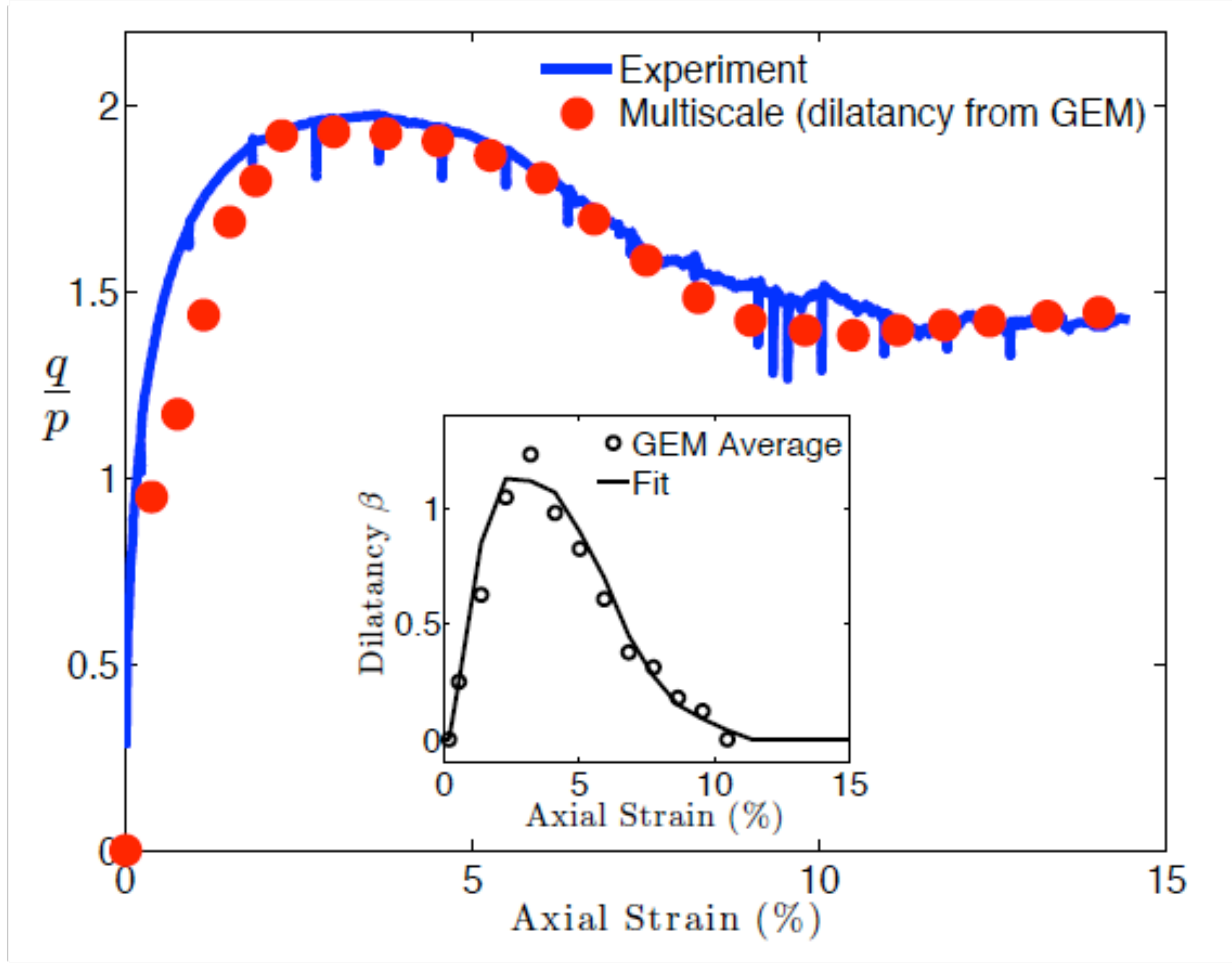
Computation of plastic parameters from DEM (micro) model



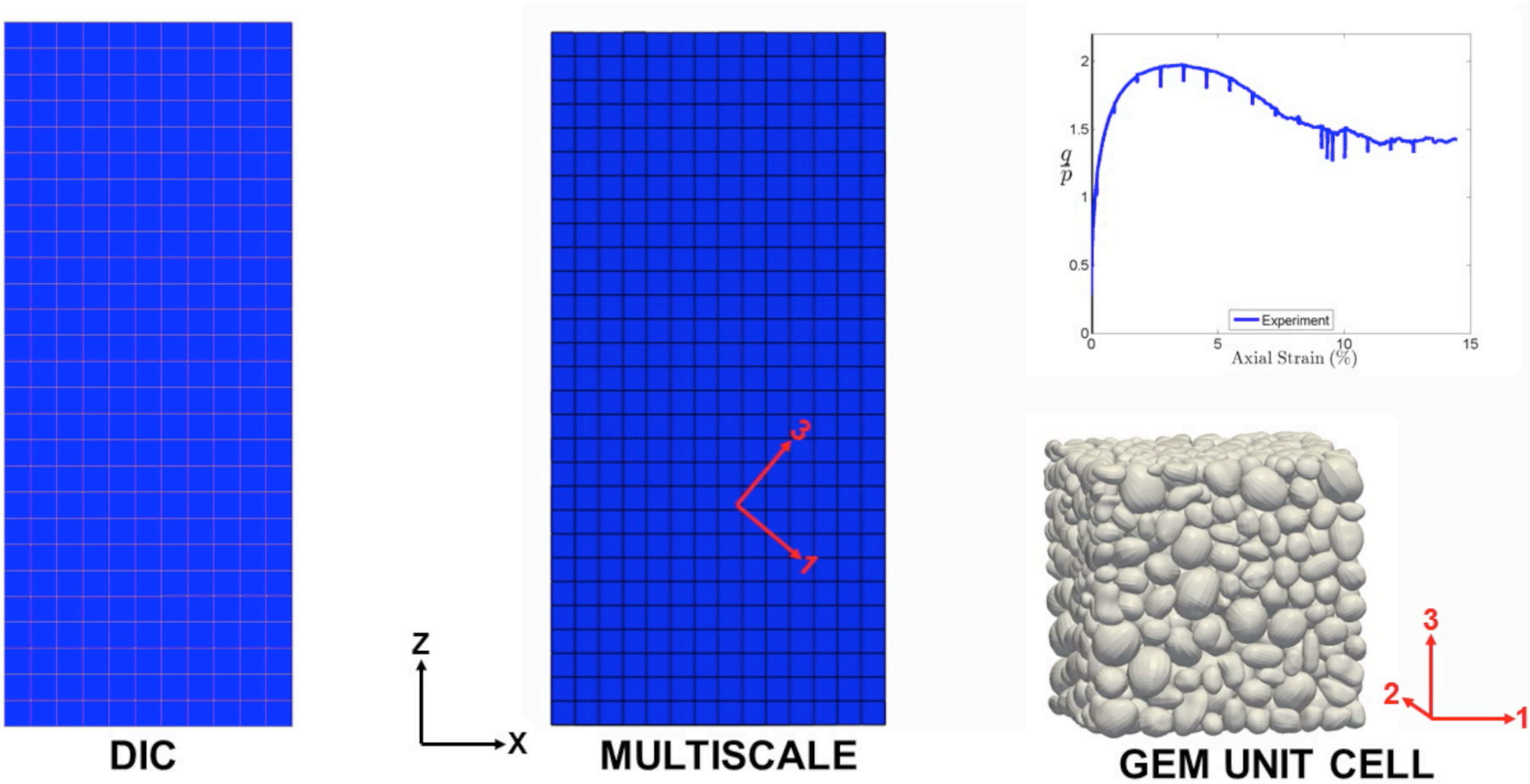
Hierarchical multiscale scheme







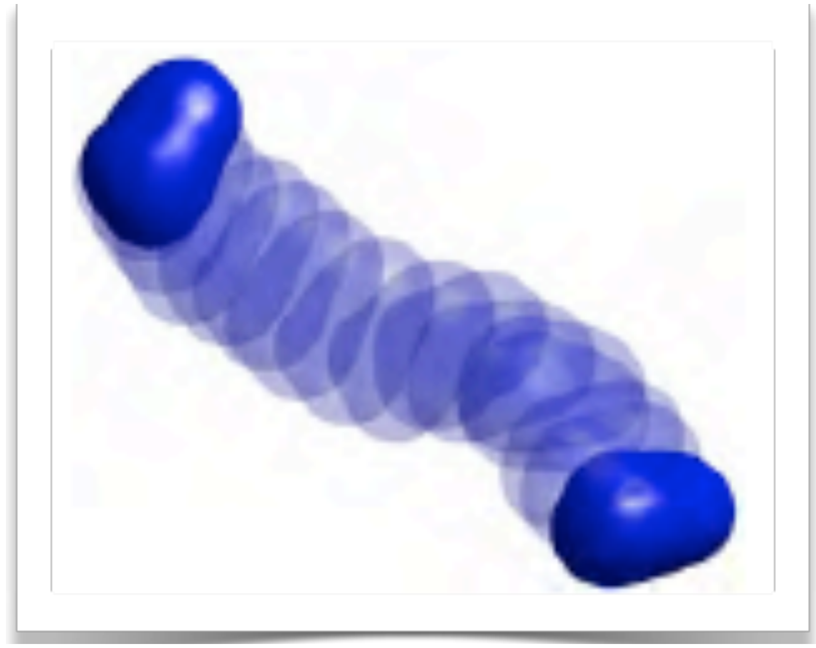
Main multiscale results

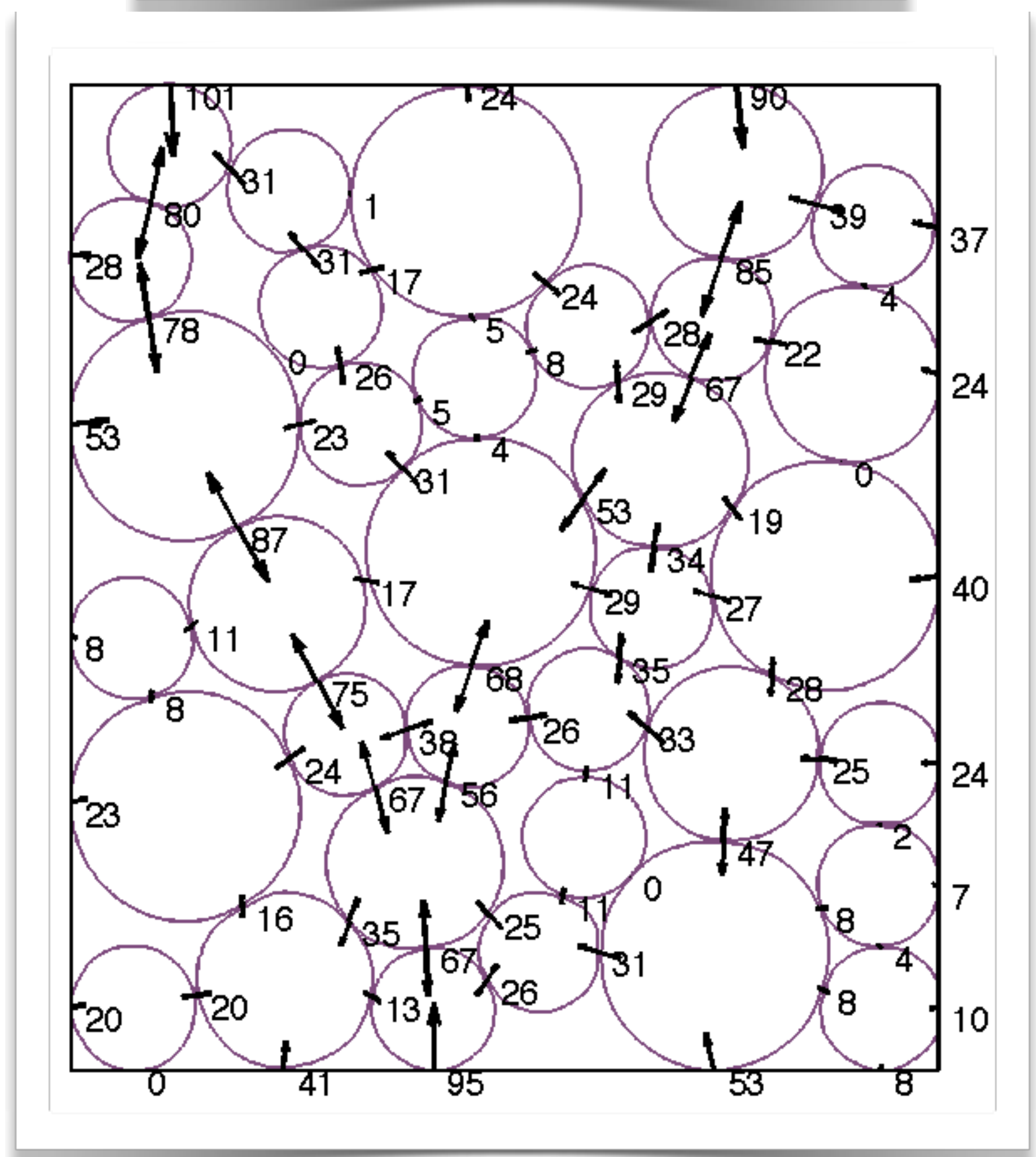


Main multiscale results

Closure 1/3

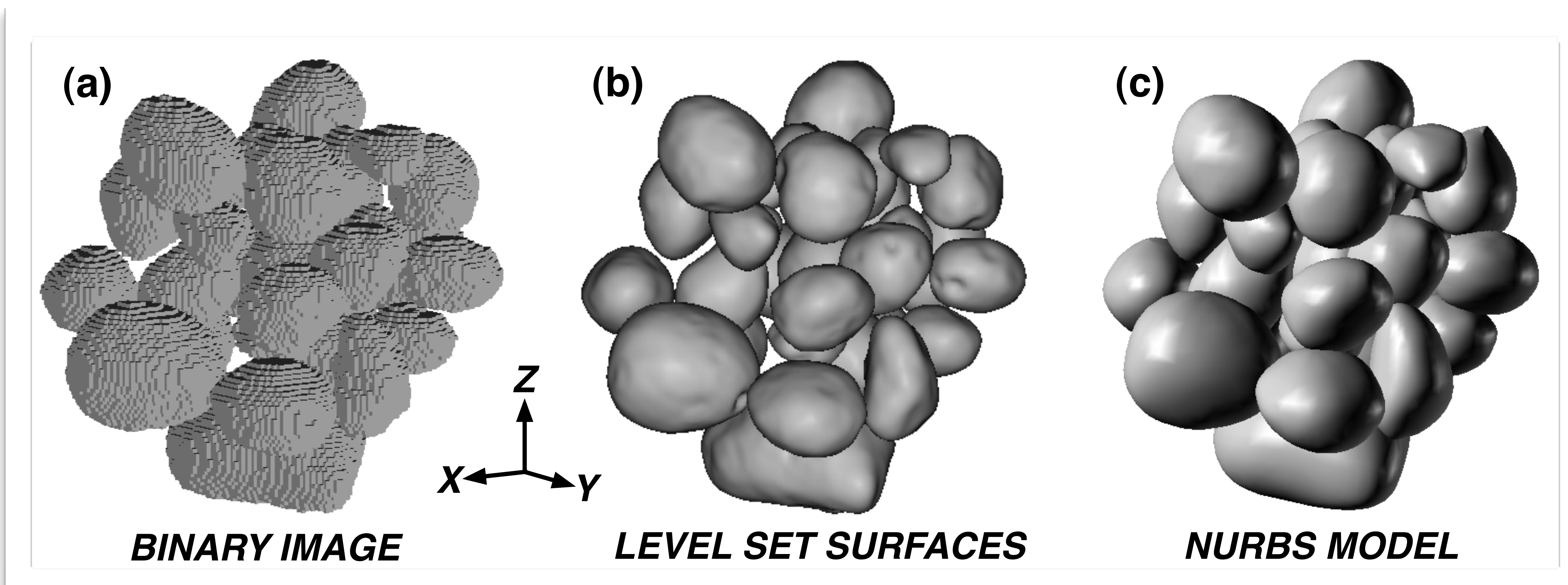
- We can now measure & compute granular BEHAVIOR: kinematics+forces





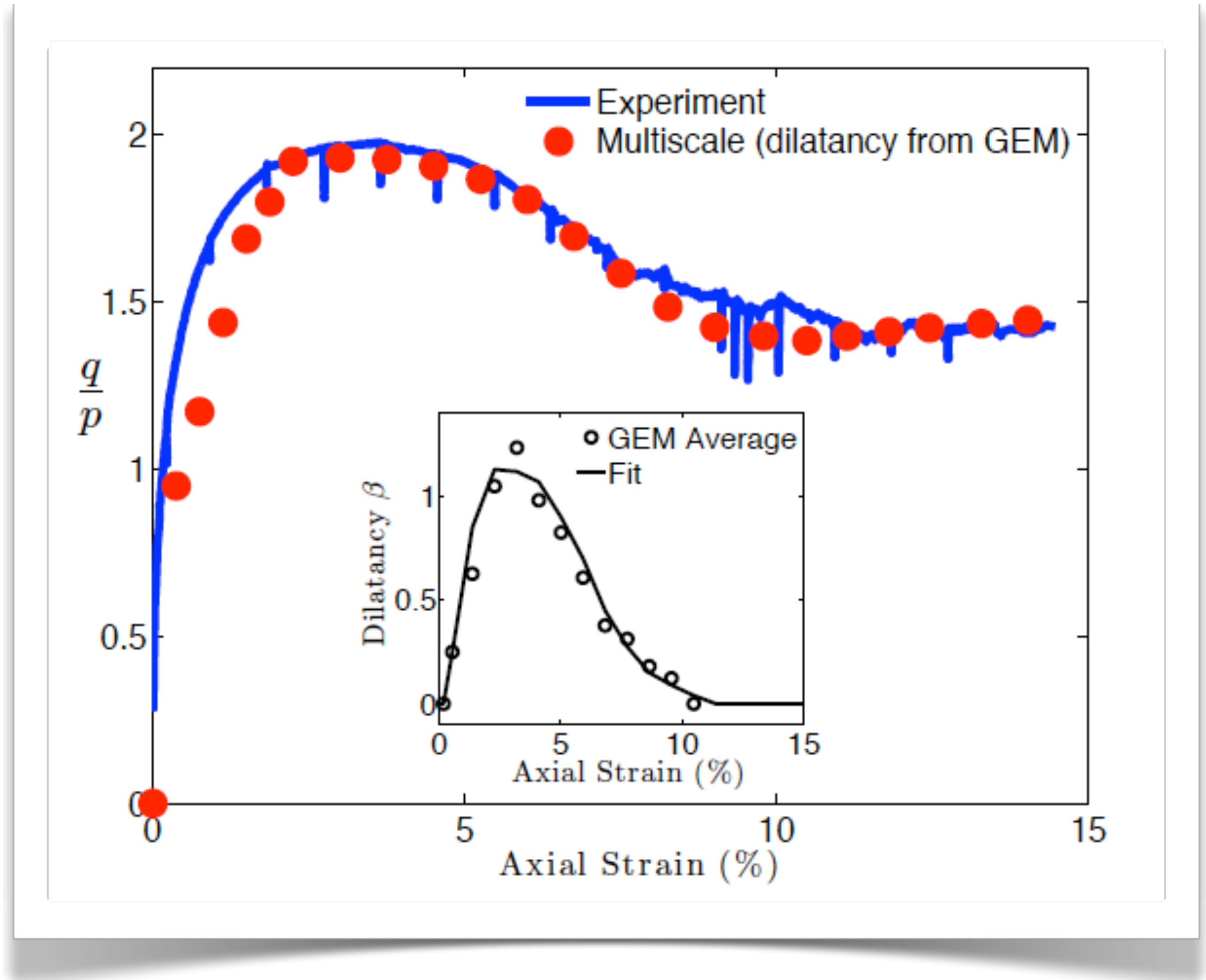
Closure 2/3

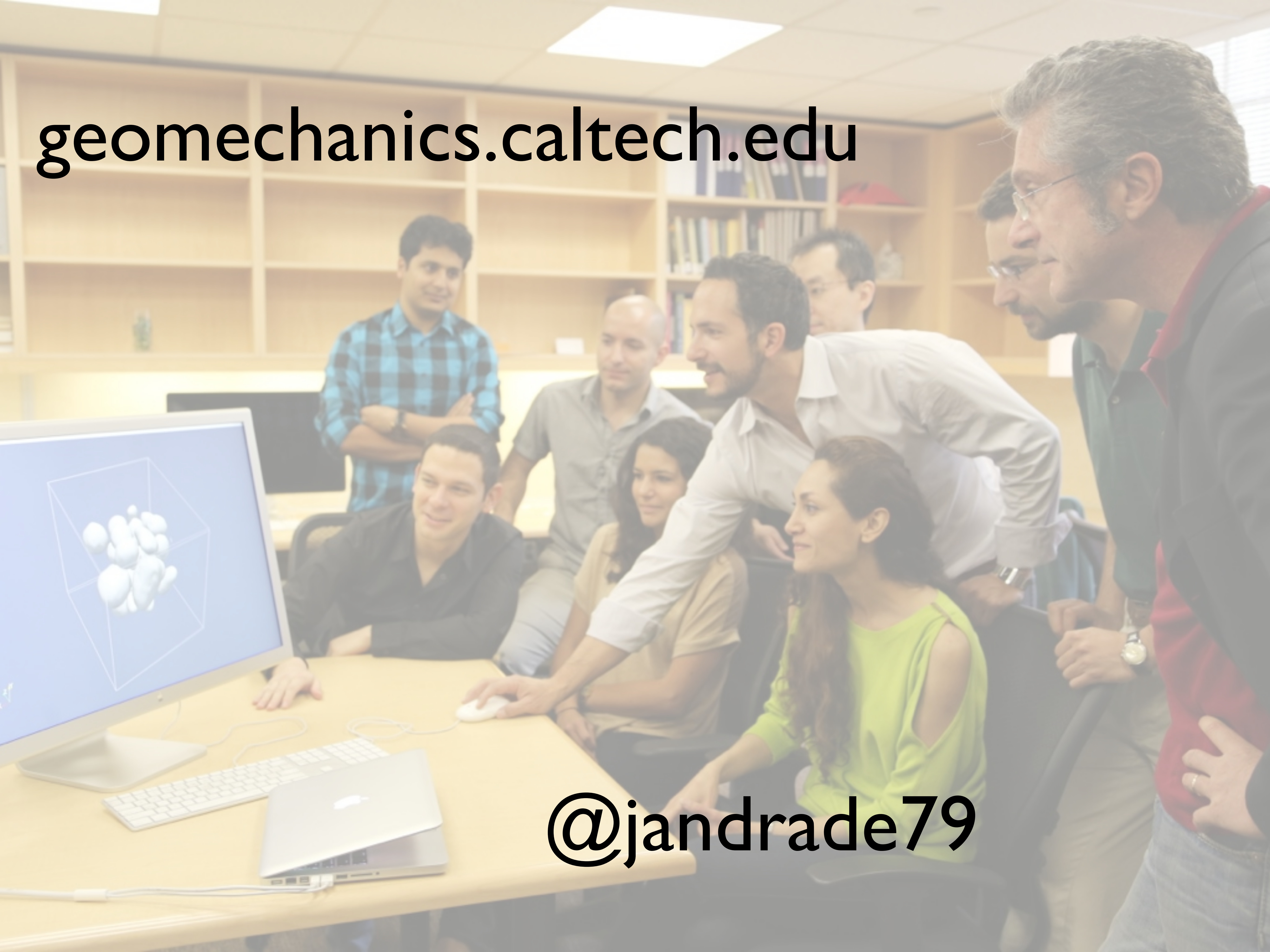
- Coupling between imaging & computational power is making progress real



Closure 3/3

- First time EVER a multiscale model has captured accurately macro behavior of REAL granular materials!




geomechanics.caltech.edu
@jandrade79