# A PROSPECT FOR TURBULENCE IN GEOMECHANICS

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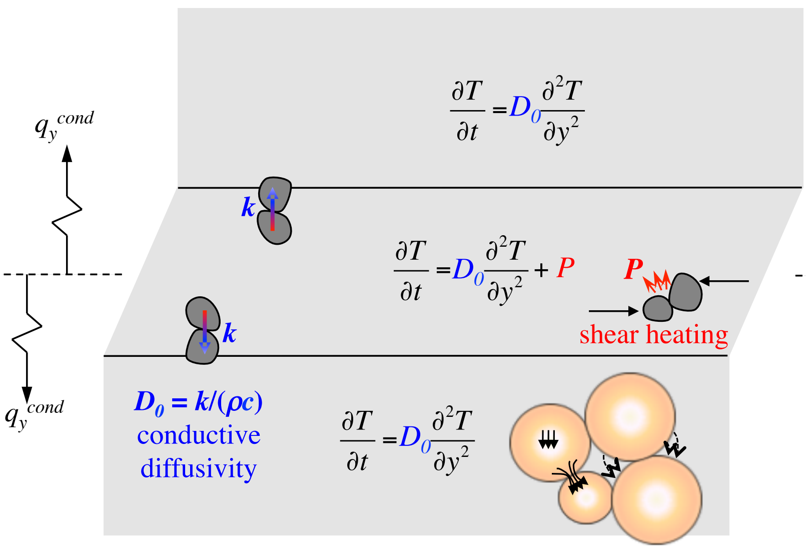
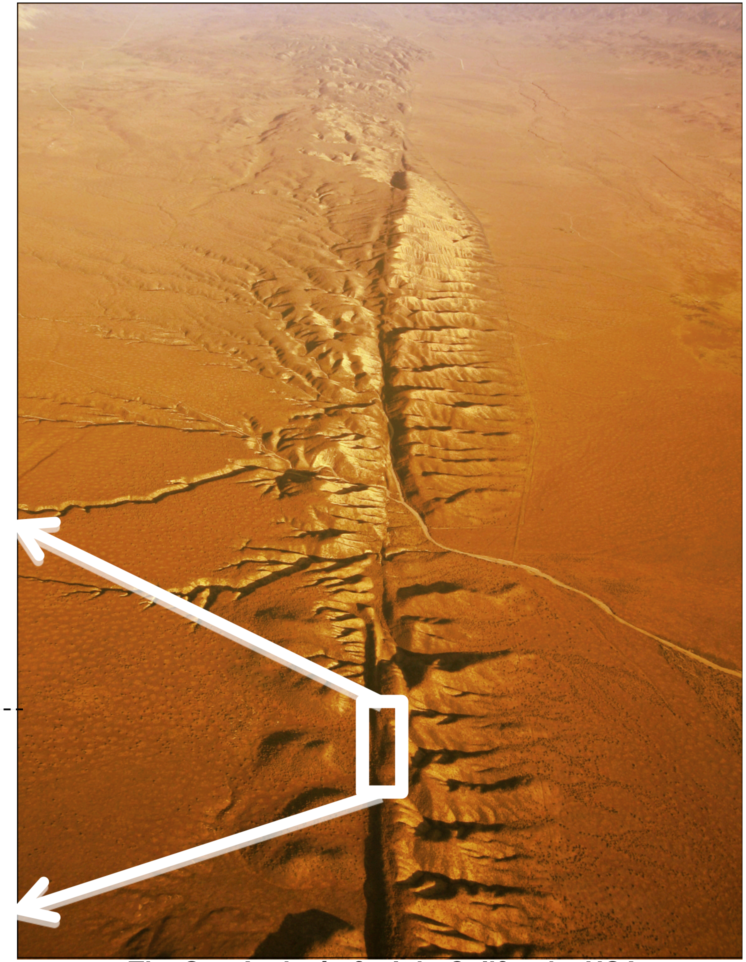
# MOTIVATION

## PARADOX OF SHEAR HEATING IN EARTHQUAKE PHYSICS

- This project originates from an attempt to quantify *shear heating* in earthquake fault gouges.
- ‘Paradox’ = anomaly of measured real heat produced vs production expected by Coulomb friction.

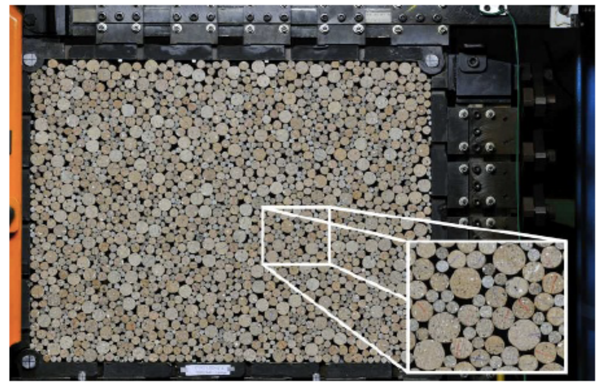


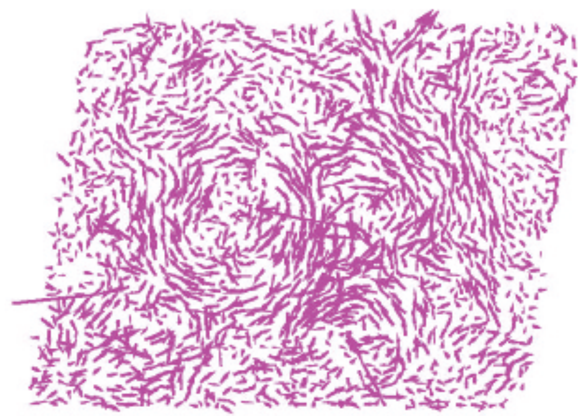



The San Andreás fault in California, USA

# MOTIVATION

## TURBULENCE IN 'QUASI-STATIC' GRANULAR MEDIA (GRANULENCE)

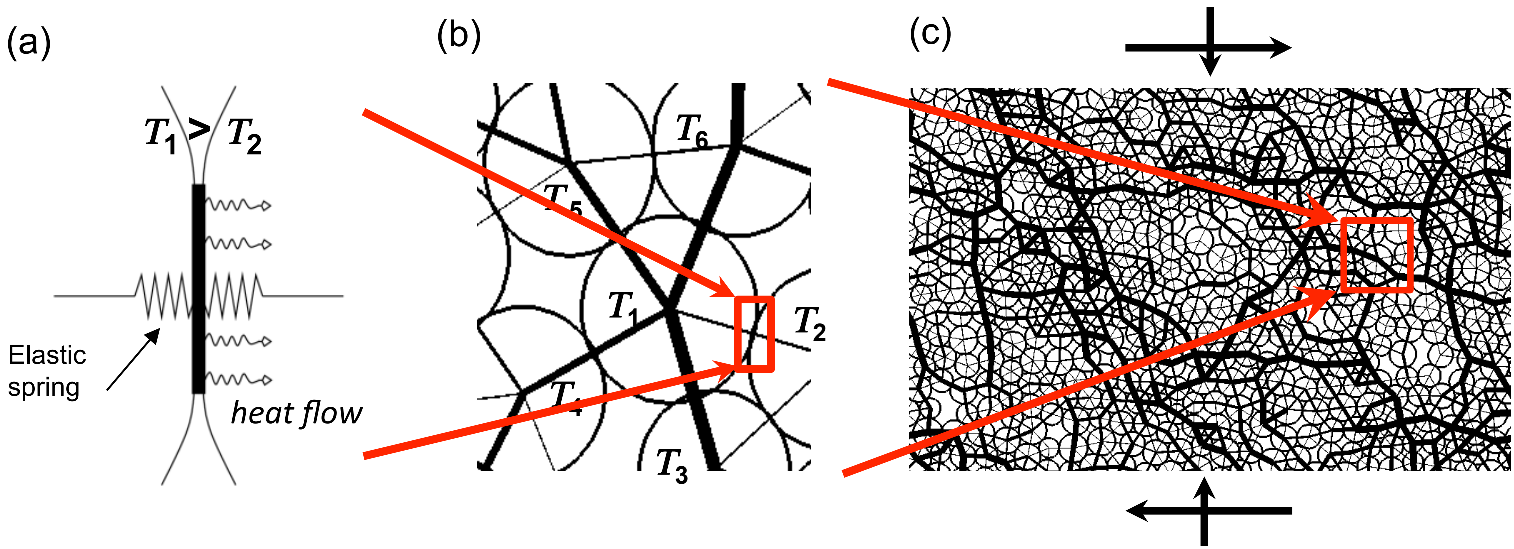




**DEM simulations:**
**Radjai-Roux, PRL 2002**

**Experimental confirmation (with 1γ2ε):**
**Richefeu-Combe-Viggiani, GLett 2012**

# THERMAL DEM




$$\varphi_{ij} = -2ak_s(T_i - T_j)$$

**Cooper-Mikic-Yovanovich, 1967**

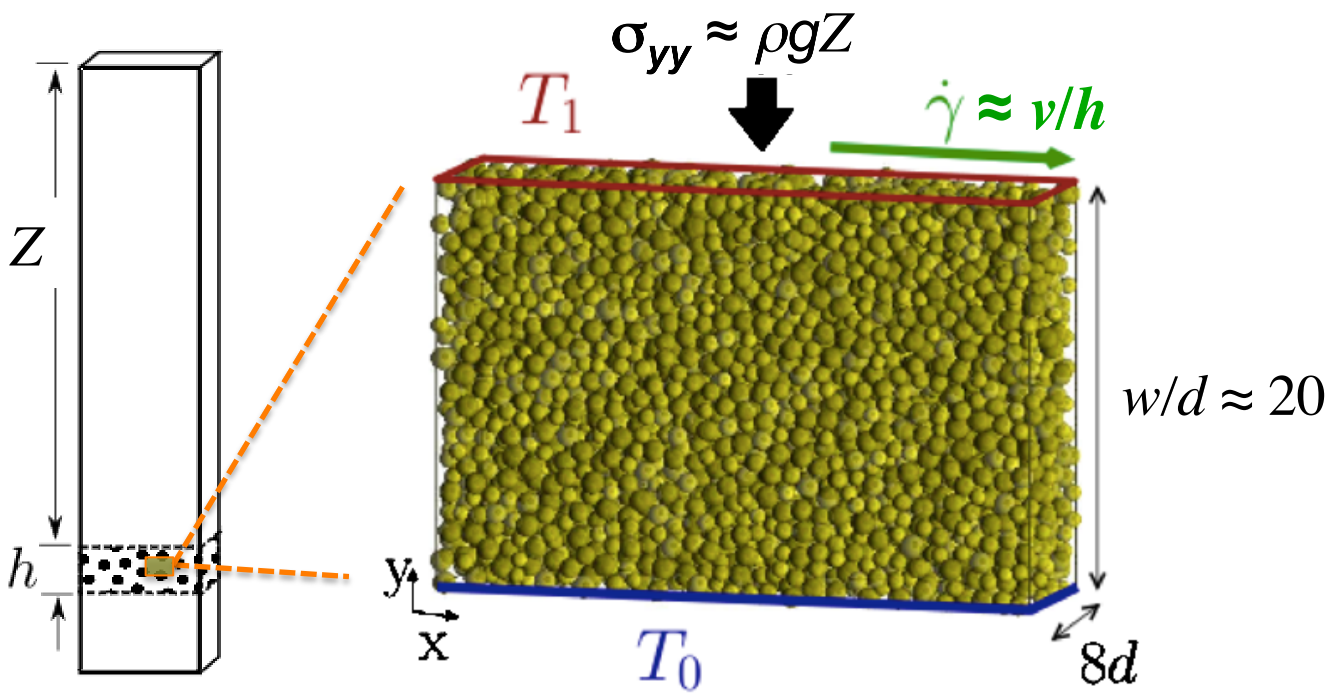
$$\dot{T}_i = \frac{1}{mc}\sum_j \varphi_{ij}$$

$$= -\frac{2k_s}{mc}\sum_j a_{ij}(T_i - T_j)$$

Vargas-McCarthy, 2001

# THERMAL DEM

## FAULT GOUGE MODEL



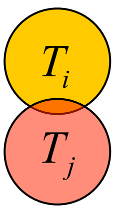

Rognon-Einav, *PRL* 2010

# TIMES & DIMENSIONLESS GROUPS

**Shear time:** $t_s = \dot{\gamma}^{-1}$

**Inertial time:** $t_i = \sqrt{m/(\sigma_{yy} d)}$

**Collision time:** $t_c = \sqrt{m/(Ed)}$




$$\frac{\partial T_i}{\partial t} = -\frac{a k_s}{mc}\left(T_i(t) - T_t\right)$$

$$T_i(t) = T_j - (T_j - T_0)\exp\left(-\frac{a k_s}{mc} t\right)$$

**Thermal time:**

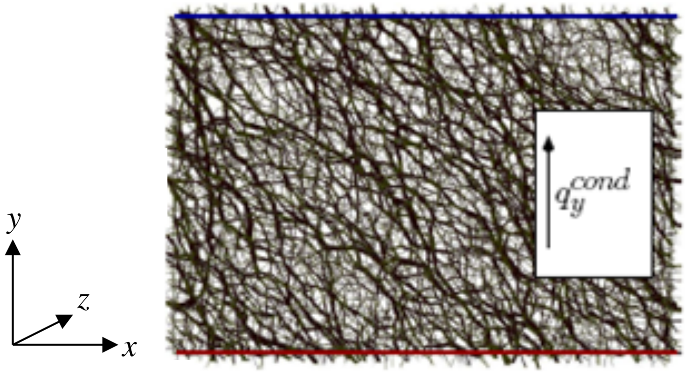
$$t_{th} = \frac{mc}{a k_s}$$

**Inertial number:** $I = t_i / t_s$

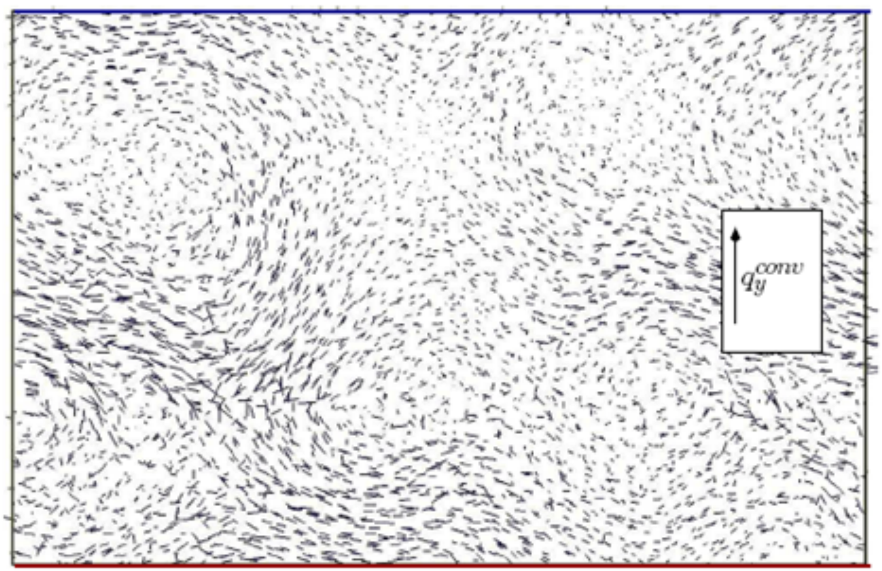
**Deformation number:** $\kappa = \sqrt{t_c / t_i}$

**Thermal number:** $\tau = t_{th} / t_i$

# CONDUCTION VS. CONVECTION




$$\vec{q}_{cond} = \frac{1}{V}\sum_{c}\phi^{c}\vec{r}^{\,c}$$




$$\vec{q}_{conv} = \frac{1}{V}\sum_{i} m_i c T_i \vec{v}_i$$

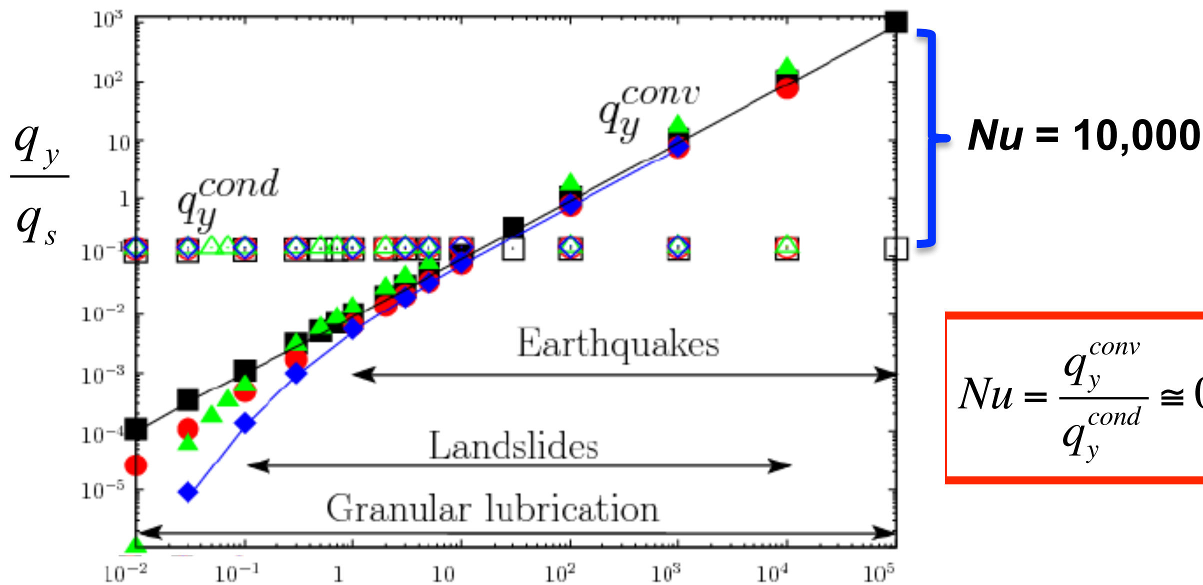
$$T_i = \nabla T_0 y_i + \widetilde{T} \quad v_{y,i} = 0 + \widetilde{v}_i$$

$$q_y^{conv} = \rho c \varphi Cov(\tilde{T}_i;\tilde{v}_i)$$

**Rognon-Einav, *PRL* 2010**

# SIGNIFICANCE OF DRY GRANULAR CONVECTION

| | $\kappa = \dfrac{\sigma_{yy}}{E}$ | $I = \dot{\gamma}\sqrt{\dfrac{m}{\sigma_{yy} d}}$ | $\tau = \dfrac{c}{k_s}\sqrt{\dfrac{m\sigma_{yy}}{d}}$ |
|---|---|---|---|
| Earthquakes | $10^{-3}$ | $0 \rightarrow 10^{-1}$ | $1 \rightarrow 10^{5}$ |
| Landslides | $10^{-6}$ | $0 \rightarrow 10^{-2}$ | $10^{-1} \rightarrow 10^{4}$ |
| Present study | $10^{-3}$ | $10^{-3} \rightarrow 10^{-1}$ | $10^{-2} \rightarrow 10^{5}$ |

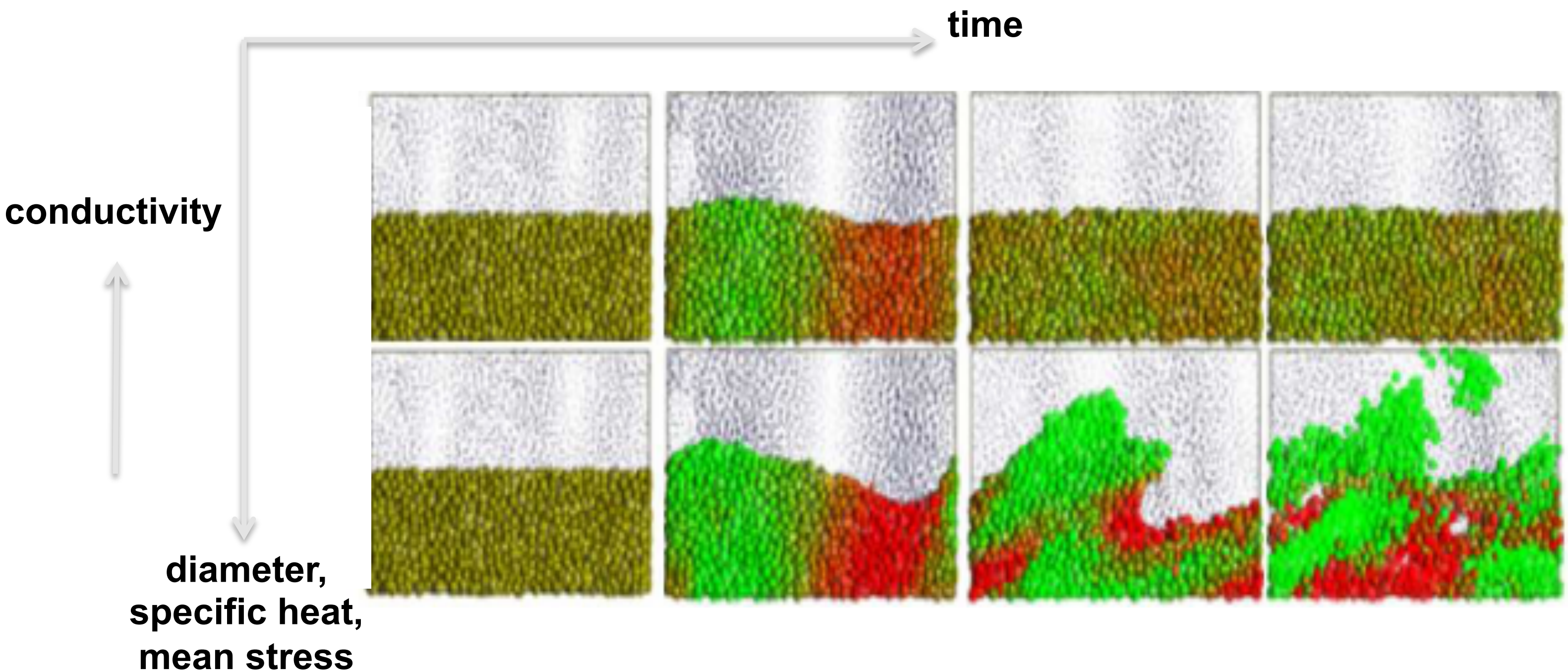


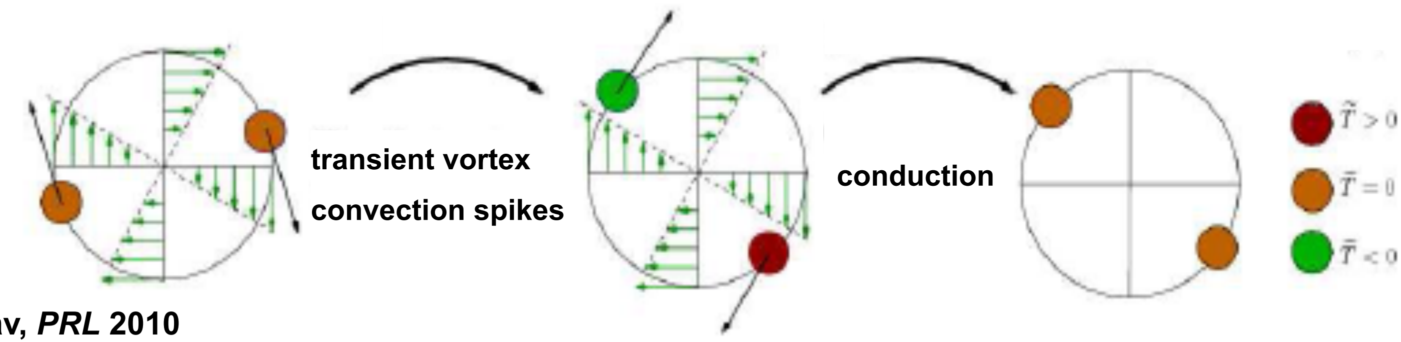

$$Nu = \frac{q_y^{conv}}{q_y^{cond}} \cong 0.1\tau$$

$$\tau = \frac{c}{k_s} d\sqrt{\rho_g \sigma_{yy}}$$

**Rognon-Einav, *PRL* 2010**

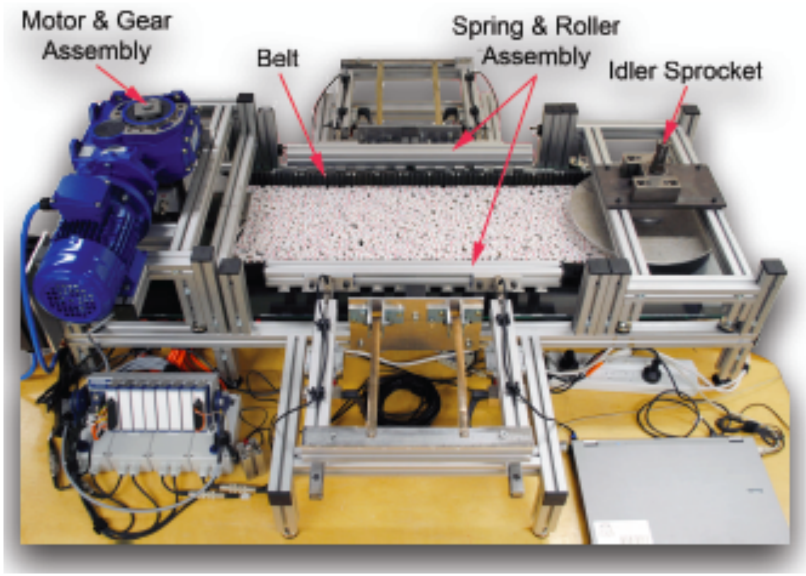
# SOURCE OF DRY GRANULAR CONVECTION

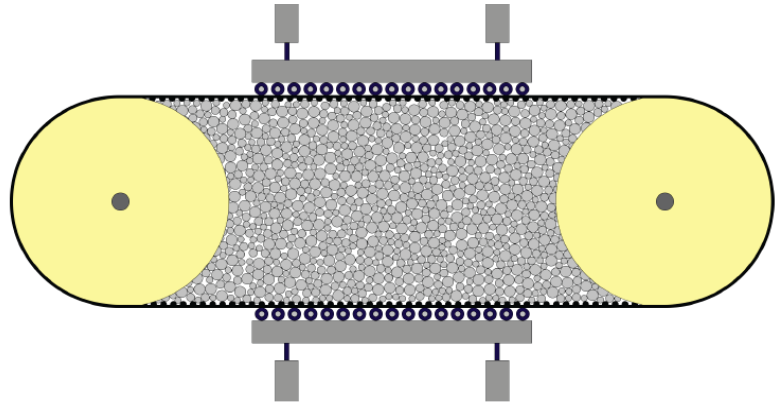


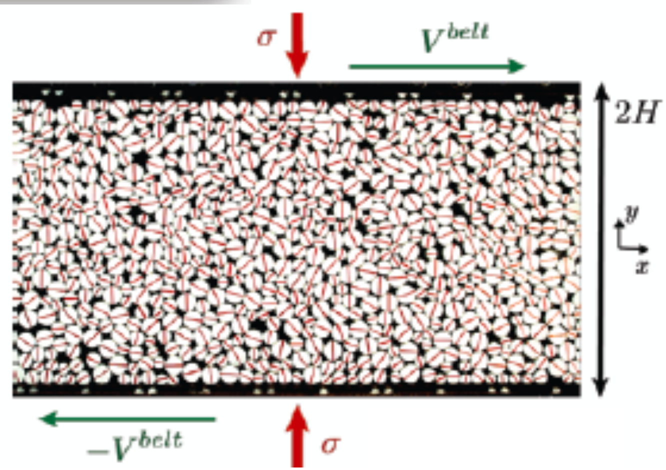




Rognon-Einav, *PRL* 2010

# STADIUM SHEAR DEVICE

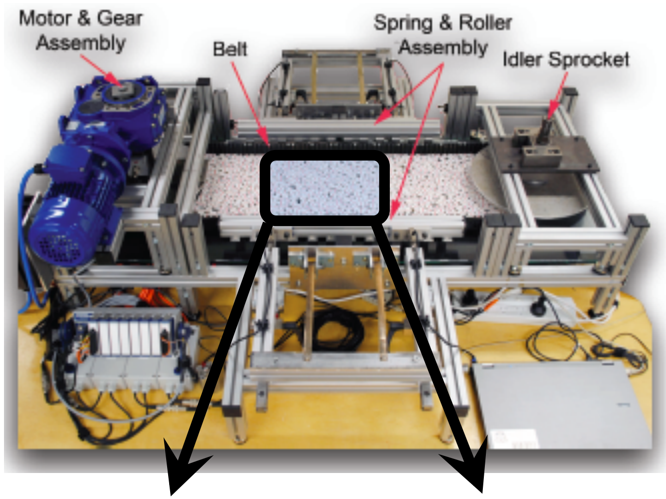


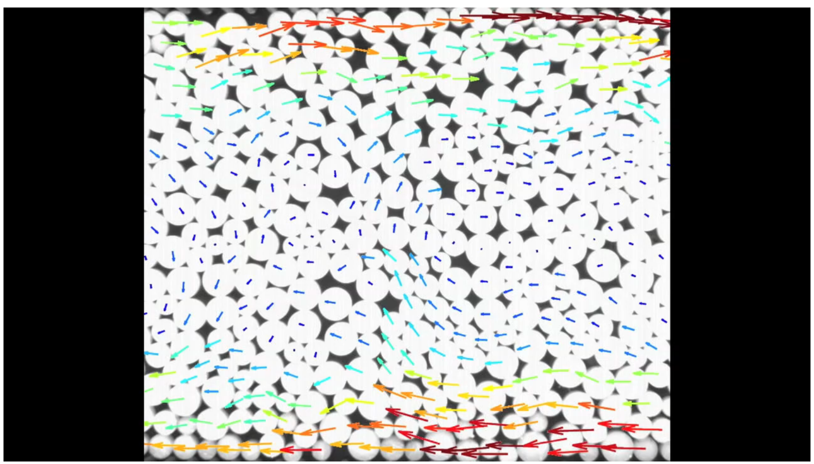


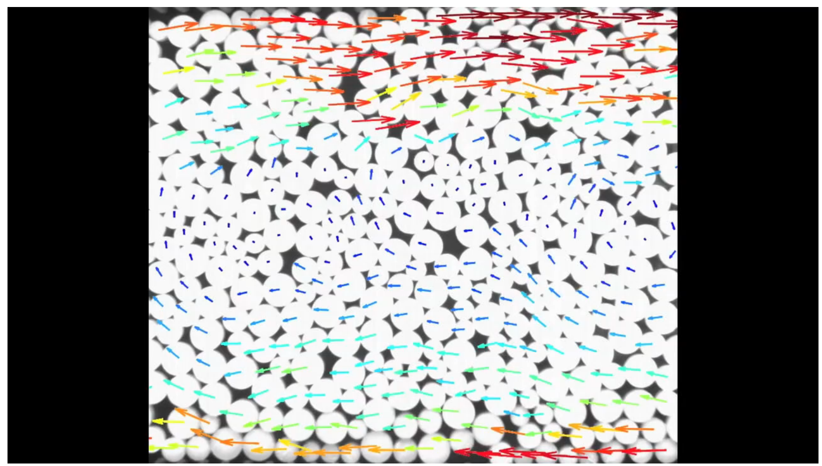

**Miller-Rognon-Metzger-Einav,**
***PRL* 2013**

# STADIUM SHEAR DEVICE



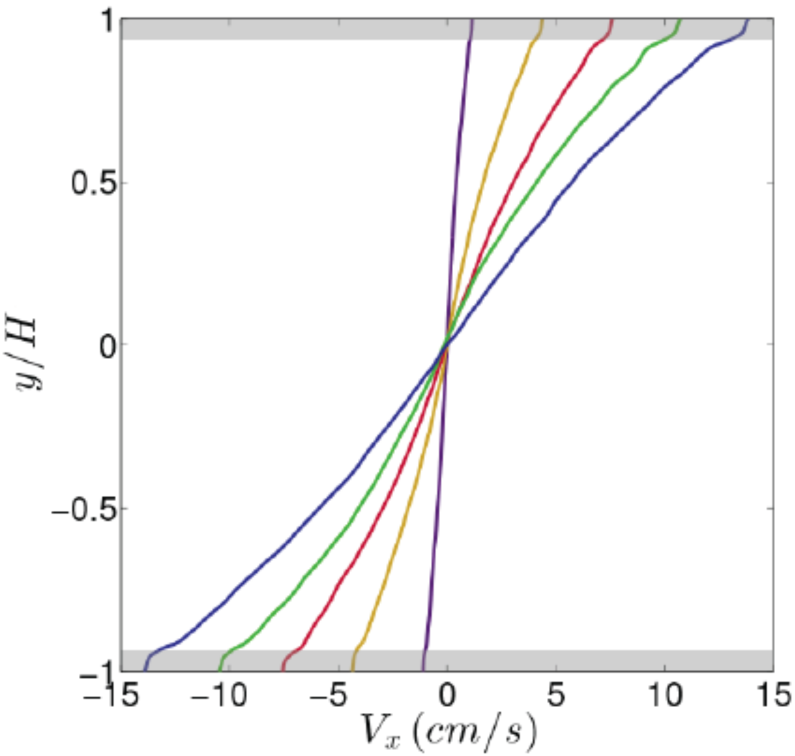

$\dot{\Gamma} = 0.08\ \mathrm{s}^{-1}$ $\dot{\Gamma} = 0.8\ \mathrm{s}^{-1}$

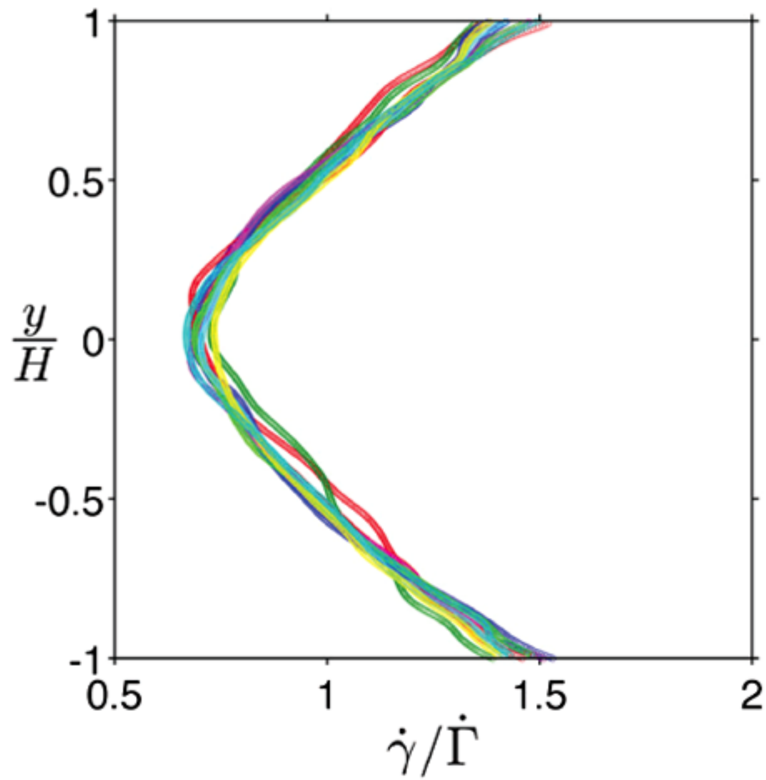




**Miller-Rognon-Metzger-Einav, *PRL* 2013**

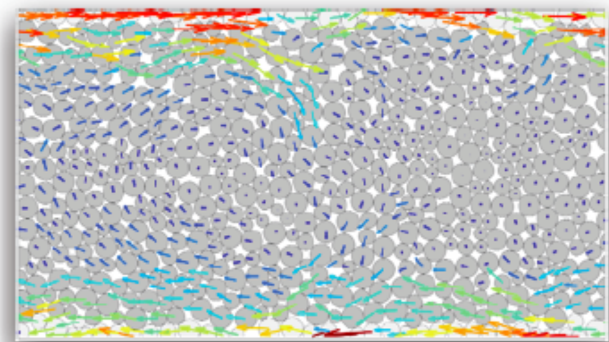
## mean velocities and strain rates



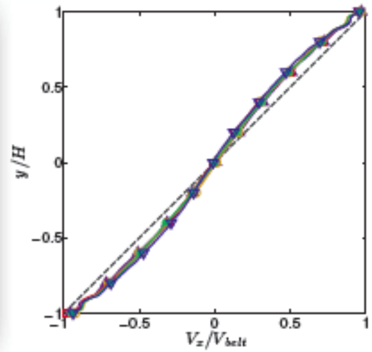




Miller-Rognon-Metzger-Einav, *PRL* 2013

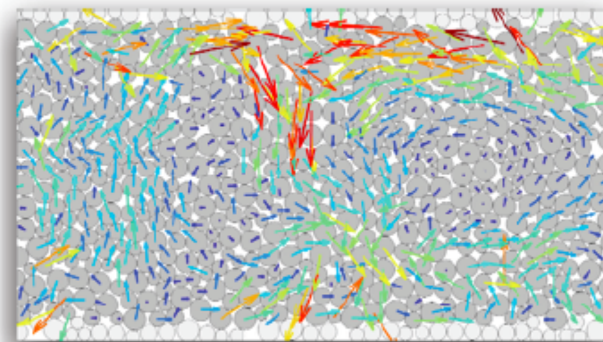
## circulation-based vortex detection



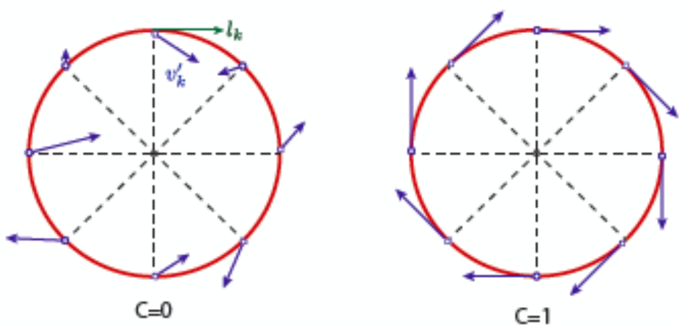
velocity field, v(x,y)



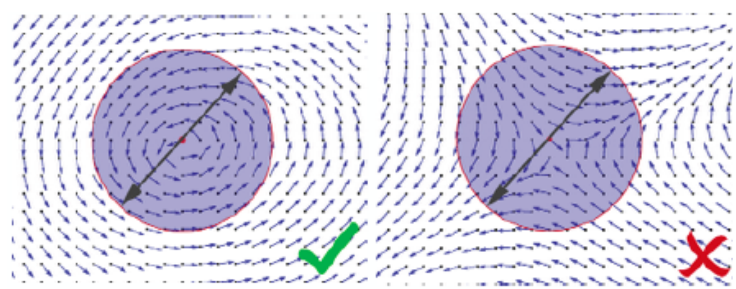

time average velocity profile



fluctuation velocities, v’(x,y)




normalised circulation, C(x,y)

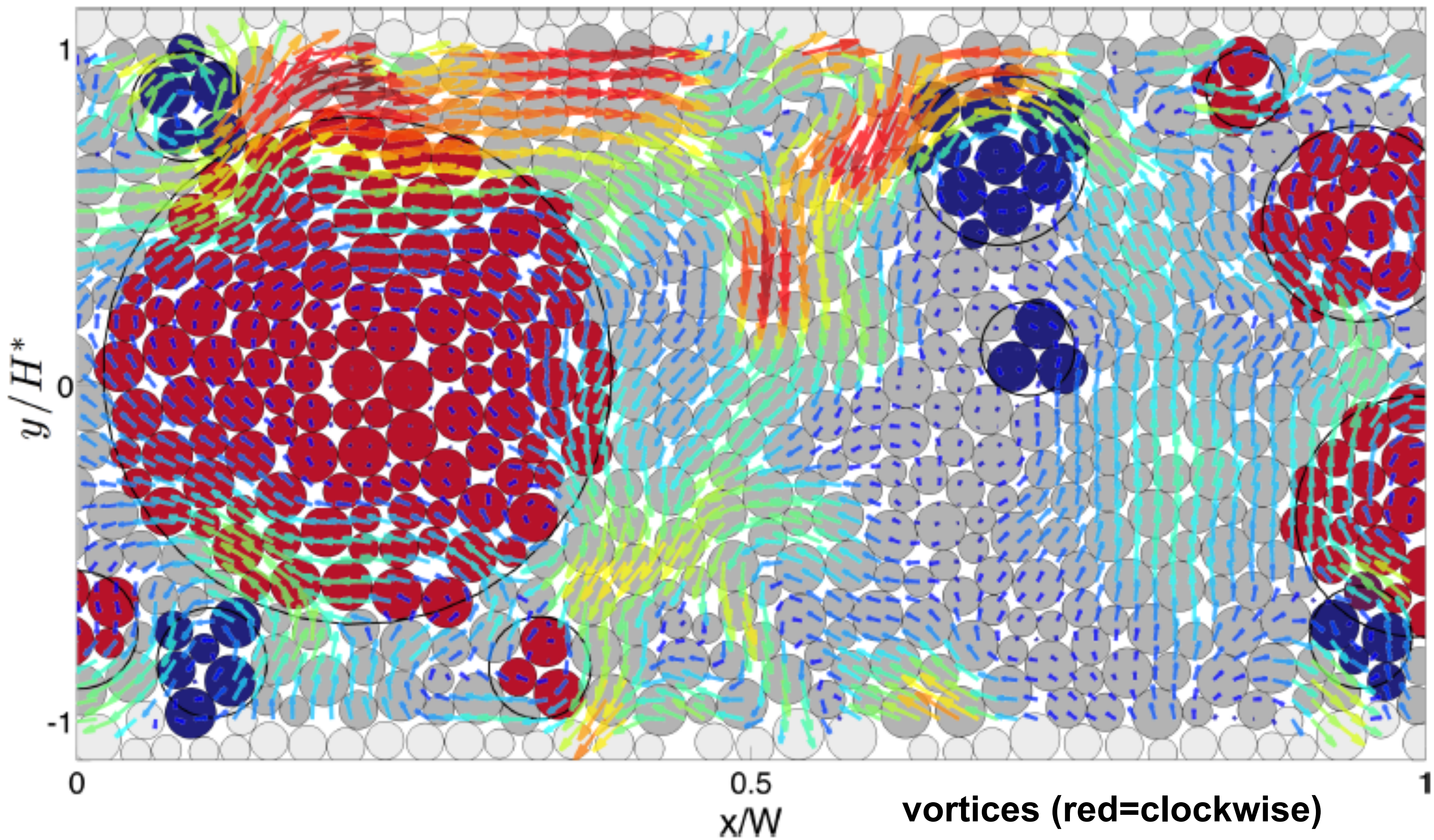


local extremum of C = vortex

Thomas Miller (PhD thesis, 2014)

## STADIUM SHEAR DEVICE

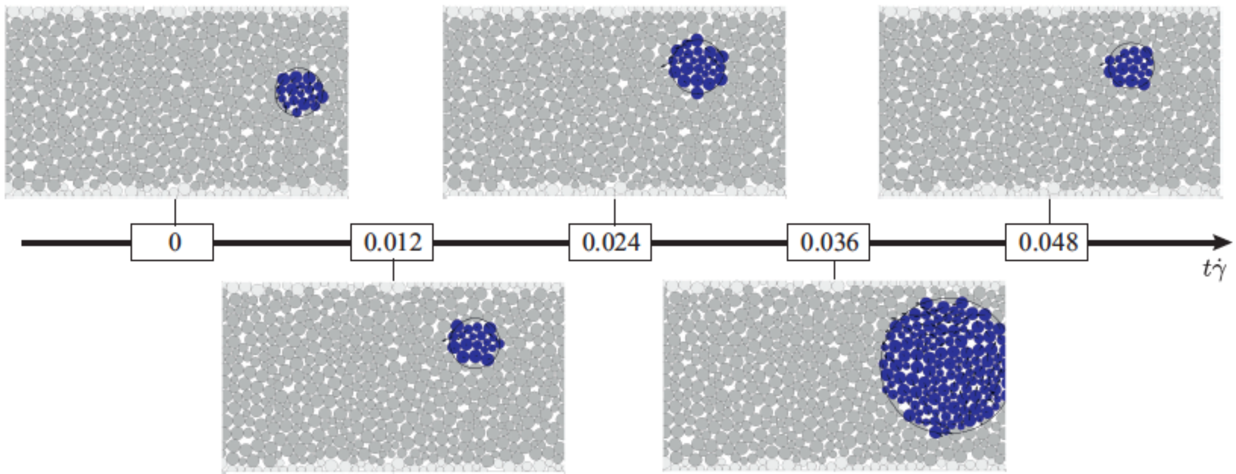
# circulation-based vortex detection

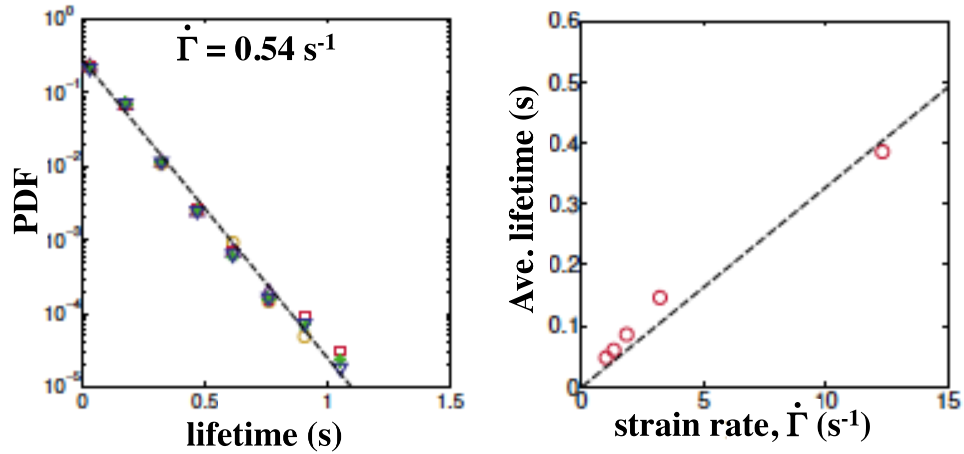



**vortices (red=clockwise)**

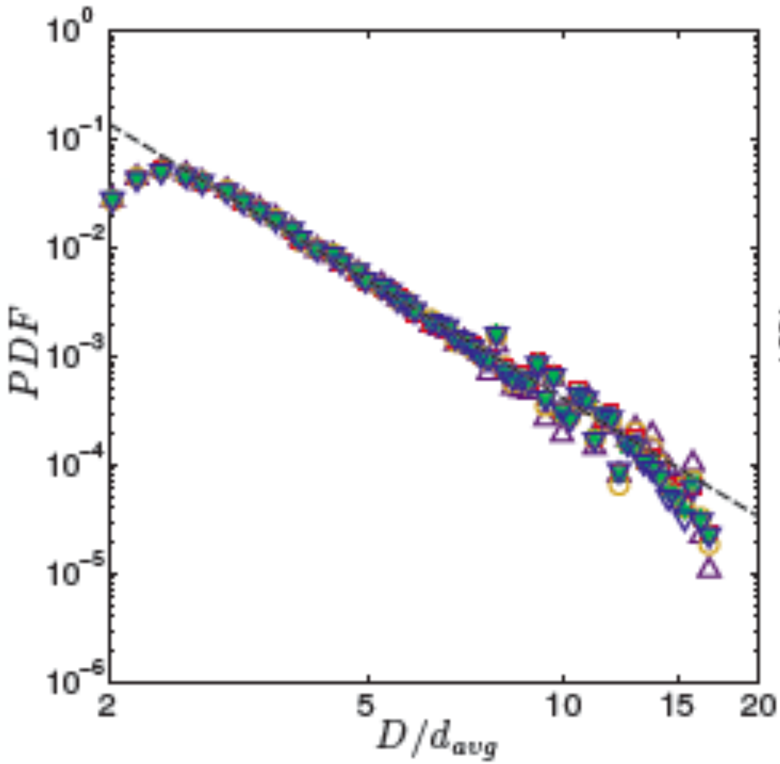
Thomas Miller (PhD thesis, 2014)

## vortex lifetime

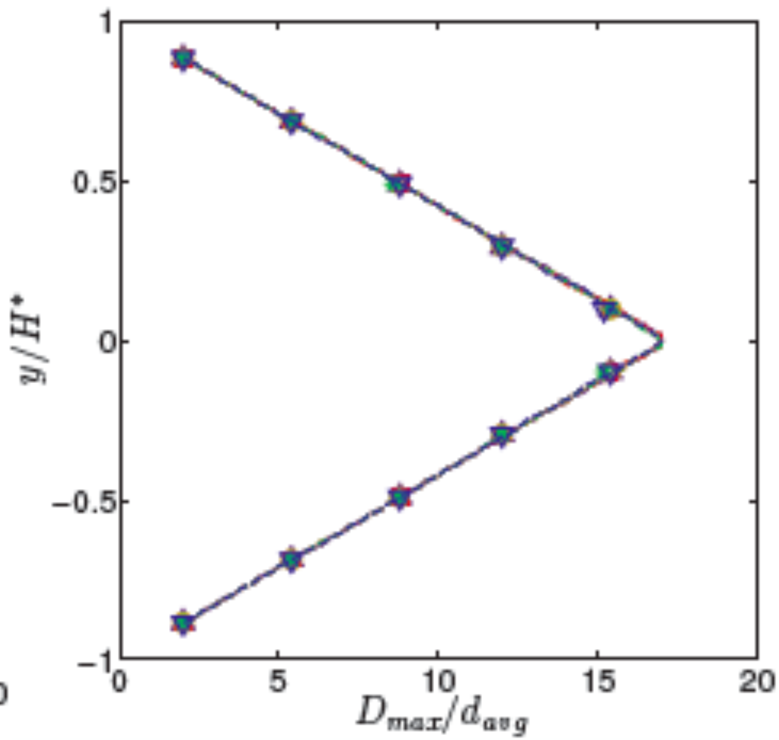


## vortex size




**vortex size distribution**




**profile of max vortex size**

# EDDY VISCOSITY

**Bagnold's scaling at steady flow suggests:** $\tau = \rho d^2 f_1(\phi)\dot{\gamma}^2$

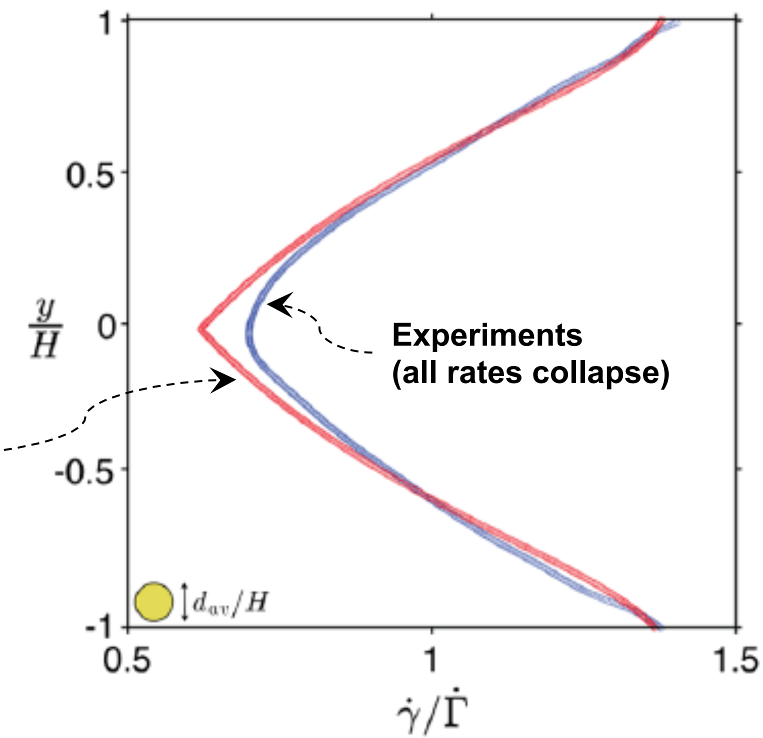
**which does not hold here since equilibrium requires $\tau$ = constant.**

**Include eddy viscosity:**

$$\tau = \rho\left(d^2 + a^2(H - |y|)^2\right) f_1(\phi)\dot{\gamma}^2$$

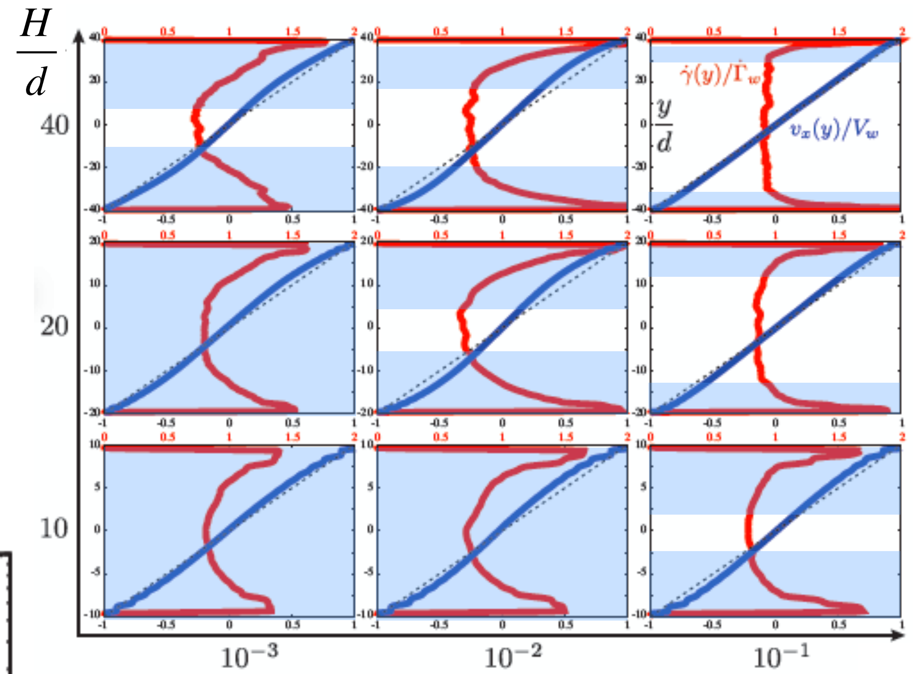
**and solve for the shear strain rate**

$$\dot{\gamma}(y) = \frac{aV^{belt}}{d\sinh^{-1}(aH/d)}\left(1 + a^2\left(\frac{H-|y|}{d}\right)^2\right)^{-\frac{1}{2}}$$

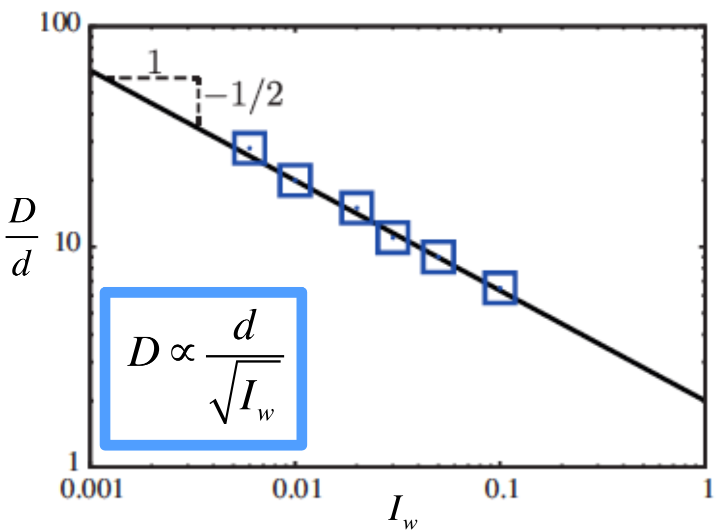



**Miller-Rognon-Metzger-Einav, *PRL* 2013**

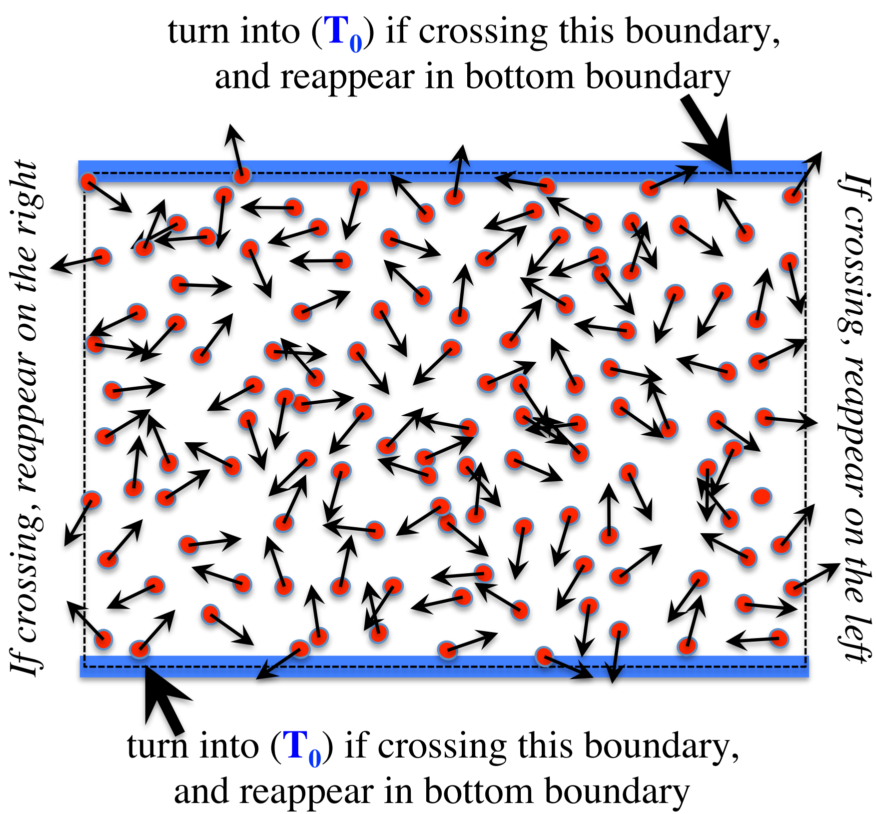
# DEM SIMULATIONS



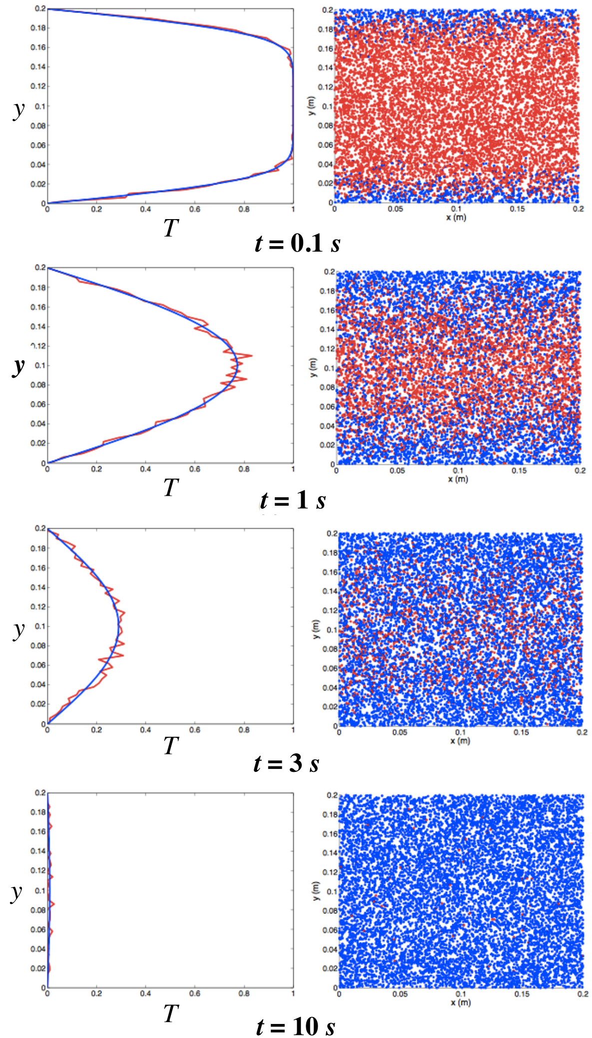

$$I_w = \frac{V_w}{H} d \sqrt{\frac{\rho_g}{\sigma_{yy}}}$$



# TRACER METHOD – MOLECULAR DIFFUSIVITY

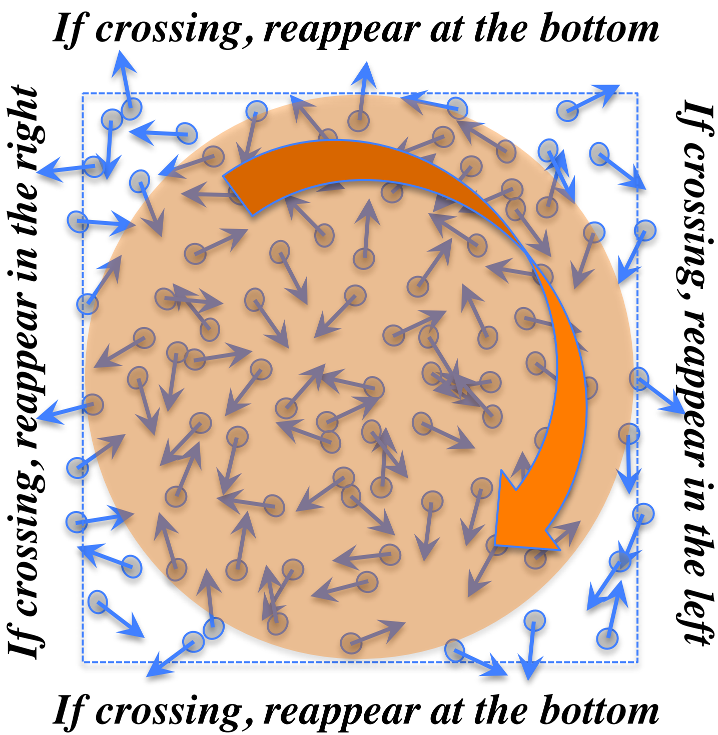


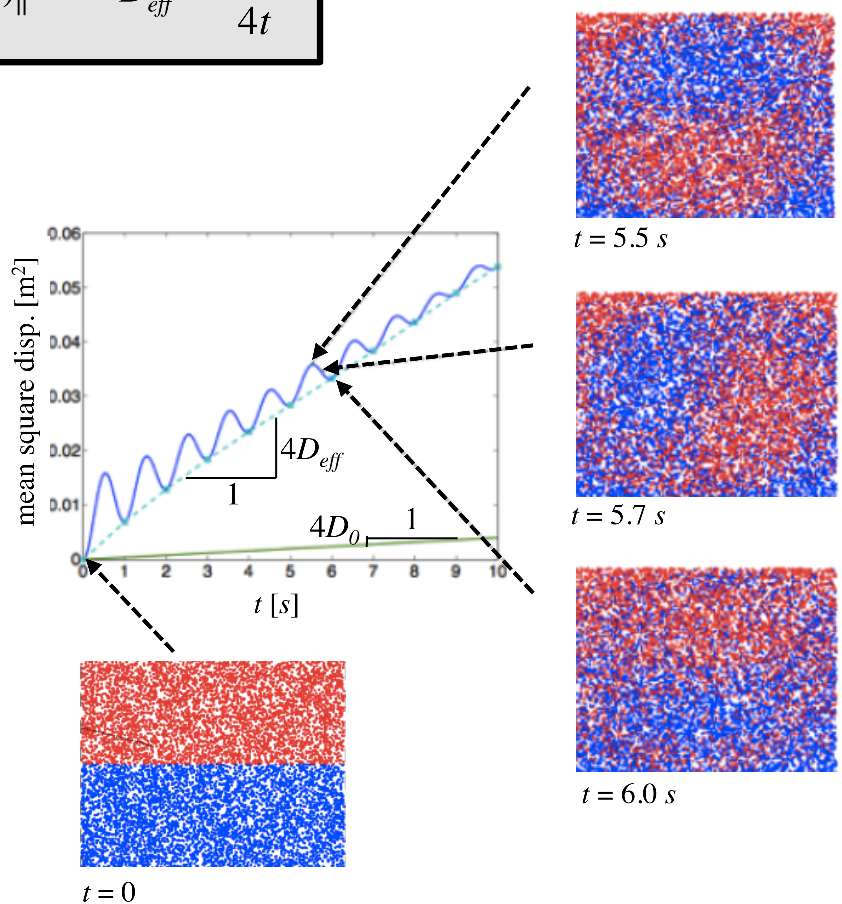

$$\boxed{v_0 = 2\sqrt{\frac{D_0}{dt}}}$$



# TRACER METHOD – 'FIXED VORTICES'

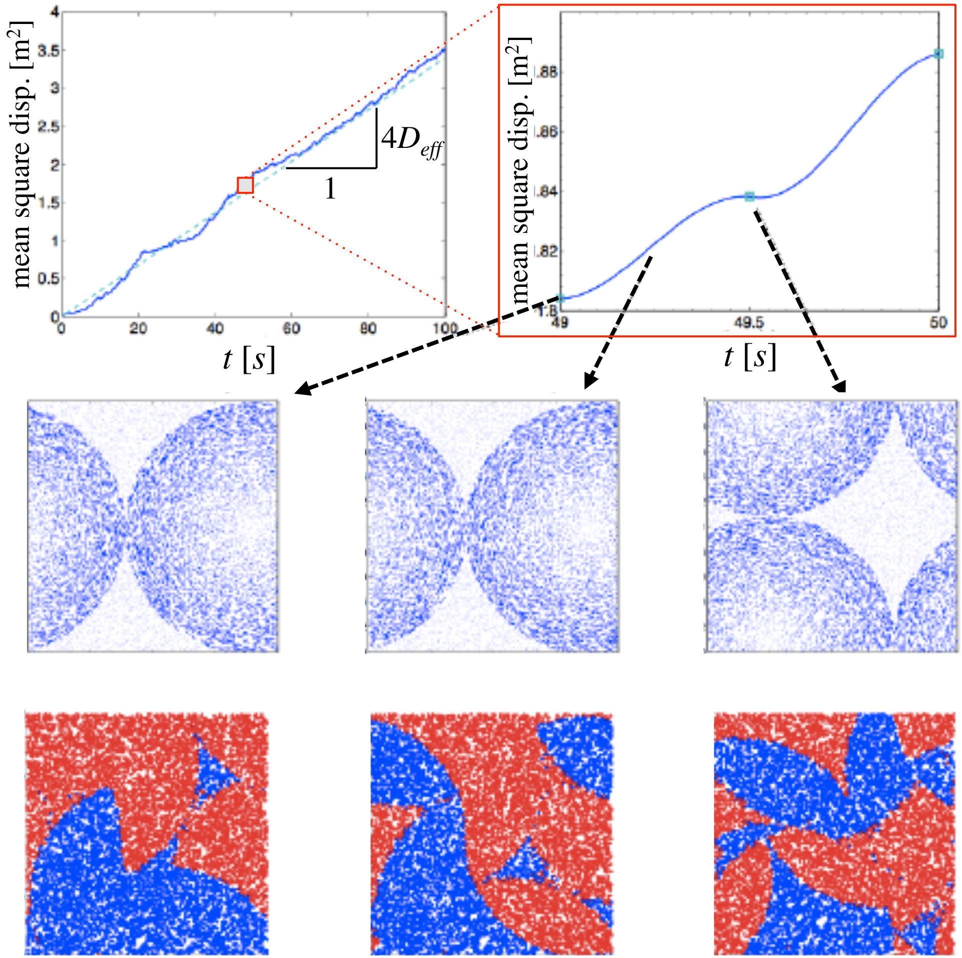
$$\lambda(t) = \frac{1}{N}\sum_{i=1}^{N}\left\|\vec{x}_i(t) - \vec{x}_i(0)\right\|^2 \qquad D_{eff} = \frac{\lambda(t)}{4t}$$



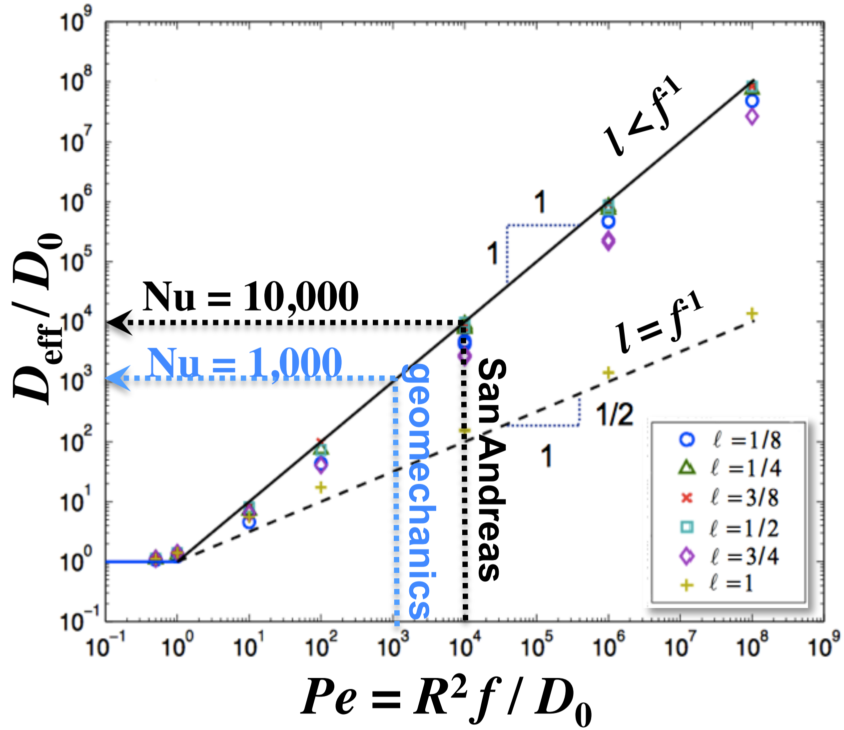

Griffani-Rognon-Metzger-Einav, *Phys Fluids* 2013

# TRACER METHOD – 'TRANSIENT VORTICES'




Griffani-Rognon-Metzger-Einav, *Phys Fluids* 2013

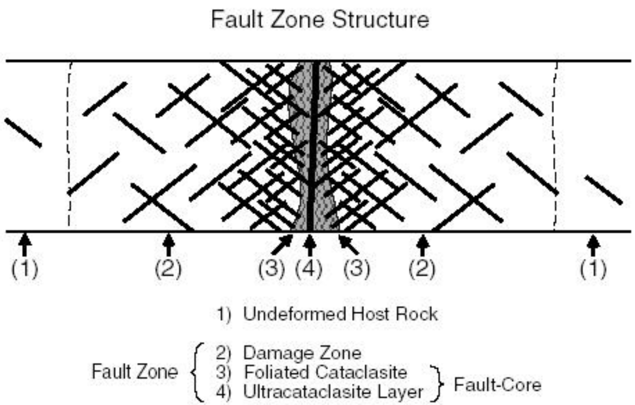
# TRACER METHOD – EFFECTIVE DIFFUSIVITY



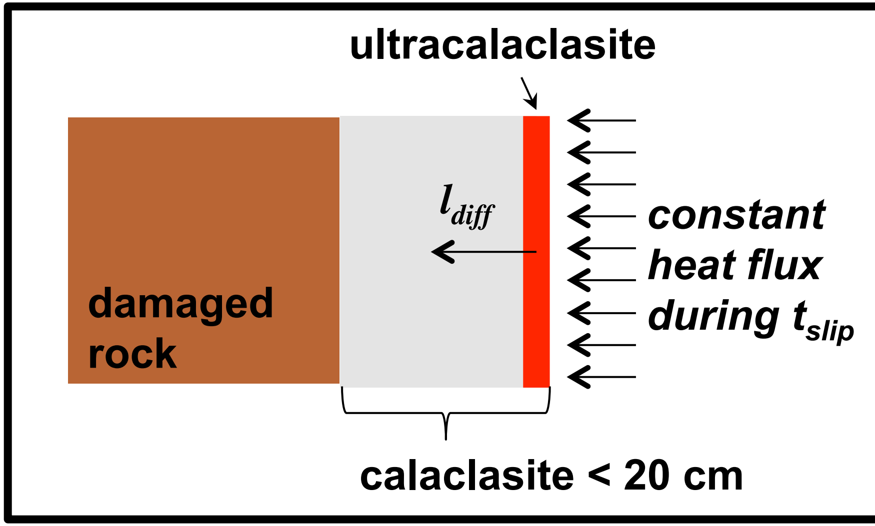

**From SSD: (1)** $R \propto d / \sqrt{I_w}$ **; (2)** $f \propto V_w / H$ **, and by definition:** $D_0 \propto k / (\rho_g c)$

**Therefore:** $\boxed{Pe \propto \frac{c}{k} d \sqrt{\rho_g \sigma_{yy}}}$ **and** $Pe \propto Nu$ **, thus explaining Thermal-DEM**

# PROSPECT IN GEOPHYSICS → MAXIMUM TEMPERATURE




**Chester, Fredrick M. et al 2004**




**without convection**

$$l_{diff}^{0} = 2\sqrt{D_0 t_{slip}}$$

$$T_{\max}^{0} \propto \frac{1}{\sqrt{D_0}}$$

**with convection**

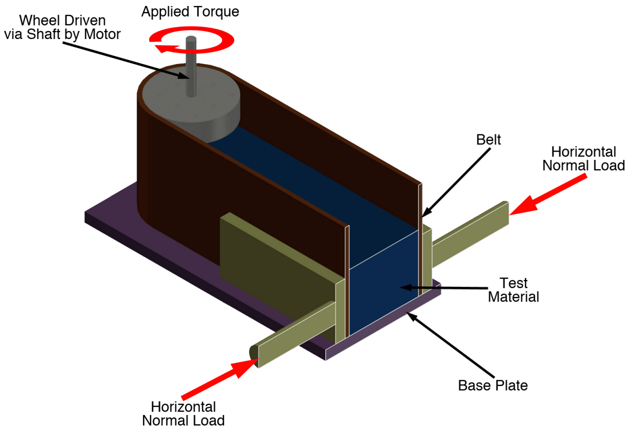
$$l_{diff}^{Nu} = 2\sqrt{(1+N_u)D_0 t_{slip}} < 10cm$$

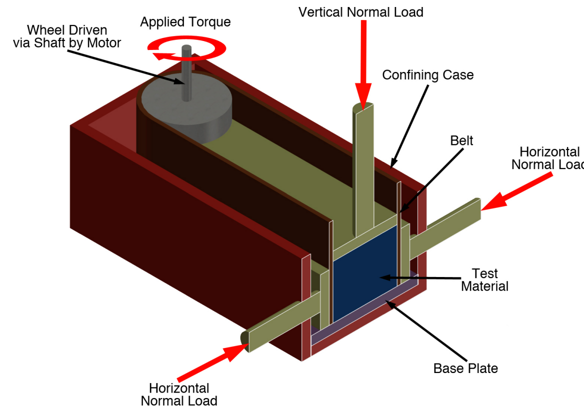
$$T_{\max}^{Nu} \propto \frac{1}{\sqrt{(1+Nu)D_0}}$$

$$\boxed{\frac{T_{\max}^{Nu}}{T_{\max}^{0}} = \frac{1}{\sqrt{(1+Nu)}}}$$

**put Nu = 10,000…**

# 3D Stadium Shear Device




## CONCLUSIONS

1. Turbulence in granular media opens new questions in geomechanics.
2. Granular heat convection can be 10,000 times that of heat conduction (even in quasi-static conditions!)
3. This has many implications, including in earthquakes, which may affect predictions and onset of activation processes
4. 10,000 or 1,000 or not, this work calls for further experimental, computational and theoretical studies (in 3D!)